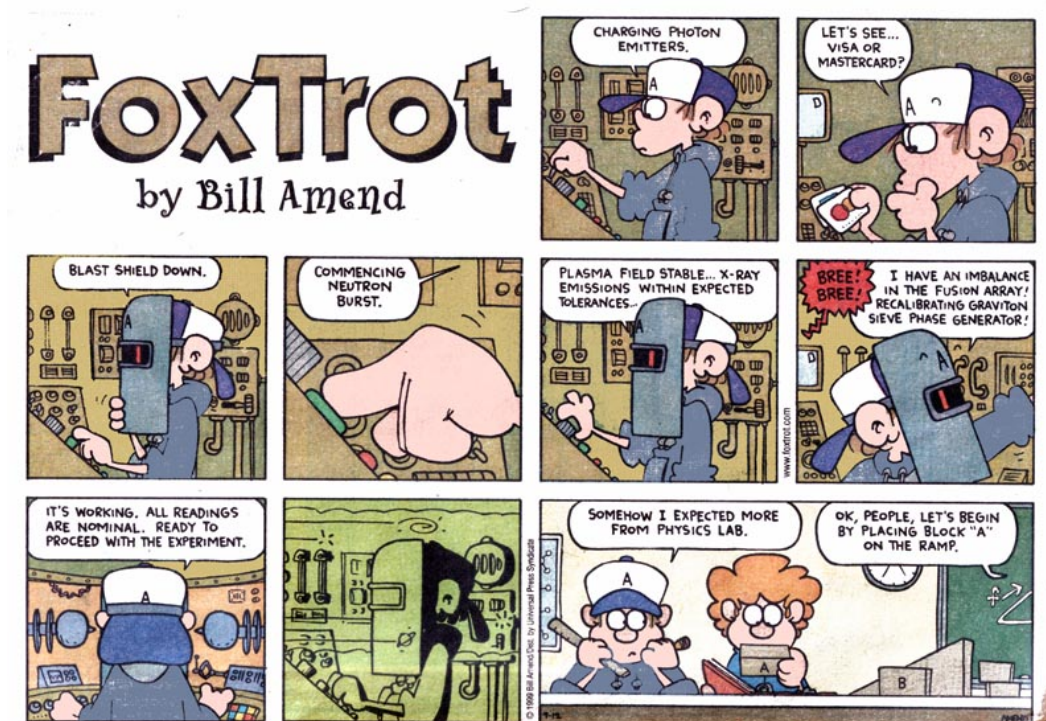# Physics 16   Laboratory Manual

Introductory Physics I: Mechanics and Wave Motion

Amherst College

Fall 2010

Contents

General Instructions

Laboratory work is an integral part of the learning process in the physical sciences. Reading textbooks and doing problem sets are great, but there's nothing like hands-on experience to truly understand physics! The laboratory sessions complement your class work. If you mentally dissociate the two and view the labs as something to be ticked off a list, you are doing yourself a great disservice, missing out on an excellent opportunity to learn more deeply.

In addition to course specific objectives, lab work is meant to develop analytic skills. Various factors may influence the outcome of an experiment, resulting in data differing noticeably from the theoretical predictions. A large part of experimental science is learning to control (when possible) and understand these outside influences. The key to any new advance based on experiment is to be able to draw *meaningful* conclusions from data that do not conform to the idealized predictions.

Introduction

These laboratory sessions are designed to help you become more familiar with basic physical concepts by carrying out quantitative measurements of physical phenomena. The labs attempt to develop several basic skills and several "higher-level" skills. The basic skills include:

1. Developing and using operational definitions to relate abstract concepts to observable quantities. For example, you'll learn to determine the acceleration of an object from easily measured quantities. One important facet of this skill is the ability to estimate and measure important physical quantities at various levels of precision.

2. Knowing and applying some generally useful measurement techniques for improving the reliability and precision of measurements, such as use of repeated measurements and applying comparison methods.

3. Being able to estimate the experimental uncertainties in quantities obtained from measurements.

The higher-level skills include the following:

1. Planning and preparing for measurements.

2. Executing and checking measurements intelligently.

3. Analyzing the results of measurements both numerically and, where applicable, graphically. This skill includes assessing experimental uncertainties and deviations from expected results to decide whether an experiment is in fact consistent with what the theory predicts.

4. Being able to describe, talk about, and write about physical measurements.

The laboratory work can be divided into three parts: 1) preparation, 2) execution, and 3) written reports. The preparation, of course, must be done before you come to you laboratory session. The execution and written reports (for the most part) will be done during the three-hour laboratory sessions.

Preparation for the Lab

You *must* do the following before coming to lab:

1. Read the laboratory instructions carefully. Make sure that you understand what the ultimate goal of the experiment is.

2. Review relevant concepts in the text and in the lecture notes.

3. Outline the measurements to be made.

4. Derive and understand the calculations for how one goes from the measured quantities to the desired results.

5. Take the pre-laboratory quiz.

6. Bring this laboratory manual and a pen to class with you.

Execution of the Lab

At the beginning of the first lab a permanently bound quadrille notebook will be given to you as your lab notebook for the semester. The cost of the notebook will be billed to your Amherst College account. The notebook is for recording your laboratory data, your analysis, and your conclusions. The notebook is an informal record of your work, but it must be sufficiently neat and well organized so that both you and an outsider can understand exactly what you have done. It is also advantageous for your own professional development that you form the habit of keeping notes on your experimental work—notes sufficiently clear and complete that you can understand them much later. Developing a good lab-taking technique requires consistent effort and discipline, skills that will be of great value in any professional career. If you become a research scientist, you will often (while writing reports or planning a new experiment) find yourself referring back to work you have done months or even years before; it is essential that your notes be sufficiently complete and unambiguous that you can understand exactly what you did then[1]. In keeping a laboratory notebook, it is better to err on the side of verbosity and redundancy than to leave out possibly important details. Appendix B in this lab manual gives instructions on how to keep a good lab notebook. You will be expected to adhere to these guidelines throughout the semester.

During the lab you will first get a brief introduction before breaking out into pairs to complete the experiment. You and your partner should follow the procedure outlined for the lab. Feel free to ask questions and discuss the lab with your partner, other student pairs, or the instructors. However, as you complete each of the steps remember to record your own notes and calculations since you will be graded individually. Things to keep in mind while completing the lab:

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[1]There have been instances in which a researcher's notebooks have been subpoenaed or used as the basis for priority claims for patents.

Do Not Erase

Never erase data or calculations from your notebook. You should always use a pen NOT a pencil to curb this habit. If you have a good reason to suspect some data is not correct (for example, you forgot to turn on a power supply in the system) or a calculation is wrong (for example, you entered the wrong numbers into your calculator), then simply draw ONE line through the data or calculation, do not erase it. Also, you must write a statement as to the reason the data is being ignored in the margin. It is surprising how often "wrong" data turns out to be useful after all.

List All Uncertainties

The stated result of any measurement is *incomplete* unless accompanied by the uncertainty in the measured quantity. By the uncertainty, we mean simply: How much greater, or smaller, than the stated value could the measured quantity have been before you could tell the difference with your measuring instruments? If, for instance, you measure the distance between two marks as 2.85 cm, and judge that you can estimate halves of mm (the finest gradations on your meter stick), you should report your results as 2.85 ± 0.05 cm. More details on uncertainties are listed in Appendix E.

List All Experimental Errors

An important (if not the *most* important) part of the analysis of an experiment is an assessment of the agreement between the actual results of the experiment and the *expected* results of the experiment. The expected results might be based on theoretical calculations or the results obtained by other experiments. If you have correctly determined the experimental uncertainty for your results, you should expect your results to agree with the theoretical or previously determined results within the combined uncertainties. If your results do not agree with the expected results, you must determine why. Several common possibilities are the following:

1. You underestimated the experimental uncertainties.

2. There is an undetected "systematic error" in your measurement.

3. The theoretical calculation is in error.

4. The previous measurements are in error.

5. Some combination of the above.

Sometimes these deviations are "real" and indicate that something interesting has been discovered. In most cases (unfortunately), the explanation of the deviation is rather mundane (but nevertheless important). Remember that small deviations from expected results have led to several Nobel prizes. So, if your results do not agree then do not "fudge" the data. Either retake the data to try to obtain a better result or list the experimental error that limited your measurement from being accurate. When you list the error, specifically state what it was, which measurements it affected, and how these incorrect measurements lead to an incorrect result.

Written Reports

You will prepare a report for each of the laboratory sessions. We will have two types: (1) short informal reports with an exit interview conducted by one of the laboratory instructors and (2) longer written formal reports.

Informal reports will, in general, focus on your in-class record of the experiment during lab time along with your answers to the questions posed in the write-up for each lab. The first part of the informal report will be an oral exit interview that you will give one of the instructors before you leave the lab. If you pass then, the instructor will initial you lab and you will be free to go! If however, the instructor feels that you have missed a key point of the lab then you will need to re-assess your data until the instructor gives you a pass and initials your notebook. Make sure to get your lab notebook initialized by one of the instructors before you leave each lab session. The second part of the informal reports will be a grade given to your lab notebooks. Every three or so labs the instructor will evaluate your lab notebooks to make sure you are following the guidelines outlined in the Appendix B.

Formal reports will be required for three of the labs (see schedule). For formal reports, you are to prepare a somewhat longer, written account of your experimental work. These reports should include a complete description of the experiment and its results. They should be typed (use a word processor) on separate sheets of paper (not in your lab notebook) and are to be turned in one week later. You should pay special attention to the clarity and conciseness of your writing. Presentation is important! If your report is not clear we will ask you to submit a revised version of the report before a grade is assigned. Guidelines for preparation of formal lab reports are included in Appendix D. While you will work in groups when you collect and analyze data in the lab, *each lab partner will write his/her own, independent lab report.*

Grading

You must complete all of the labs to pass this class. We set the labs up only for the week they are to be performed, so if you have to miss a lab because of illness, family difficulties, or other legitimate reasons, please alert your instructor (when possible). All make-up labs for pre-arranged absences should be arranged before the end of the second week.

Your lab grade will be worth 24% of your total class grade or 240 points out of 1000 points. The nine pre-laboratory quizzes will be worth 10 points each for a total of 90 points. The nine informal reports that you will write in your lab notebook as you complete the lab will also be worth 10 points each for a total of 90 points. Remember, to get credit for the informal reports you must pass the exit interview and have each lab initialized. Finally, each of the three formal lab reports will be worth 20 points for a total of 60 points.

Intellectual Responsibility

Discussion and cooperation between lab partners is strongly encouraged and, indeed, essential during the lab sessions. **However, each student must keep a separate record of the data,**

must do all calculations independently and must write an independent lab report. It is strongly advised that students do not communicate with each other, in person or electronically, once the writing process has begun. Specific questions concerning the writing of reports should be directed to the instructor or teaching fellow. In addition, laboratory partners are expected to share equally in the collection of data. The sharing of drafts of reports, use of any data or calculations other than one's own, or the modeling of discussion or analysis after that found in another student's report, is considered a violation of the statement of Intellectual Responsibility.

We wish to emphasize that intellectual responsibility in lab work extends beyond simply not copying someone else's work to include the notion of scientific integrity, i.e. "respect for the data." By this we mean you should not alter, "fudge," or make up data just to have your results agree with some predetermined notions. Analysis of the data may occasionally cause you to question the validity of those data. It is always best to admit that your results do not turn out the way you had anticipated and to try to understand what went wrong. You should **never** erase data which appear to be wrong. It is perfectly legitimate to state that you are going to ignore some data in your final analysis if you have a justifiable reason to suspect a particular observation or calculation.

1 Experimental Background: Measurements and Uncertainties

1.1 Introduction

The heart of any experiment is making measurements. All measurements are subject to uncertainty; no matter how precise the instrument that is used or how careful the experiment is done. Therefore it is important to evaluate in some way the size of the uncertainty in a measurement, and if possible, minimize that uncertainty. Often you will hear the word "error" used interchangeably with uncertainty. However, in the context of science, error or uncertainty does not mean "mistake", it simply is a number that defines the reliability of a measurement. More details on defining and calculating uncertainties can be found in Appendix E. Also, an excellent book on error is John R. Taylor's *An Introduction to Error Analysis*, 2nd ed.

In this lab we will explore the different techniques used to evaluate the uncertainty or error in a measurement, and how that error effects the outcome of an experiment. We will also talk about various statistical methods to average data and to calculate statistical significance. While this might seem straight forward, this is probably the most important lab you will do this semester! The reason is that because any field that involves data will also have to involve error and statistics. So if you would like to enter medicine, pharmaceuticals, geology, education, politics, engineering, chemistry, etc., you will need to know this. Even as an ordinary citizen listening to the news or advertisements you will be amazed at how much data will be tossed around at you, with gasp, no errors, no definitions of averages, and no insight into the sample size or makeup. A great example of bad statistics is from *How to Lie with Statistics* by Darrell Huff (published in 1954). In it Huff remarks on the label on the side of a tooth paste: "Users report 23% fewer cavities with Doakes' tooth paste the big type says. You could do with twenty-three percent fewer aches so you read on. These results, you find, come from a reassuringly 'independent' laboratory, and the account is certified by a certified public accountant. What more do you want?" As Huff later describes the tooth paste company didn't lie about the statistic, it's just that they only had 12 people in the test. With a low sample size, like 12, you can get large deviations from expected behavior. The same thing happens if you flip a quarter 12 times, you are supposed to get 50% heads, but sometimes you might get 80% heads. Does that prove that quarters always come up 80% heads? No, definitely not. The moral is that error and statistics are important, so let's learn how to do them correctly.

1.1.1 Estimating Uncertainties

The Uncertainty in an Analog Scale The simplest measurement one can make is comparing an object to a scale like a ruler. Here we will look at analog scales, or scales where the values aren't displayed digitally. Look at Fig. 1 below. If we assume the edge of the arrow is aligned with the edge of the ruler, *and* the edge of the ruler represents the zero of the ruler's scale, then what is the length of the arrow? Since the tip of the arrow lies between the markings on the ruler, we have to *interpolate* the position of the arrow tip. A reasonable value for the length of the arrow may be 5.5 cm. How do we choose a reasonable *estimate* of the uncertainty? I stress word estimate

because uncertainties are just that, estimated values. One way to estimate the uncertainty is by considering what is *not* a reasonable value of the length of the arrow. Most people would agree that the length of the arrow is less than 5.9 cm and greater than 5.1 cm. Can we make a better estimate of the length of the arrow? Perhaps less than 5.8 cm and great than 5.2 cm. We continue this process until we reach a point where we are *uncertain* what the length is. Our first estimated was 5.5 cm, but someone else may see 5.6 cm or 5.4 cm. The result is there is an uncertainty in the actual value of the length of the arrow. The range of values that represent a reasonable estimate of the length is the uncertainty in the length. In this particular case ±0.1 cm is our estimate.
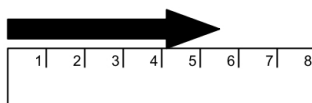
Figure 1: Ruler measuring length of arrow

The Uncertainty in a Digital Scale   Throughout the semester we will use digital scales to measure quantities, the most common case is a digital balance to measure mass. What is the uncertainty in a digital measurement? Unlike an analog scale, there is no way to interpolate the value from a digital display. What you see is what you get. The rule of thumb for digital displays is that an uncertainty of ±1 unit in the last digit is a good estimate. So if a digital balance reads 1.923 kg, then the uncertainty is ±1 gram.

Absolute vs. Relative Uncertainties   How is the uncertainty of a measurement reported? There are conventionally two ways to express the uncertainty of a measurement: absolute and relative uncertainty. The absolute uncertainty is expressed in this form:

$$x_{measured} = x_{best} \pm \Delta x \tag{1}$$

The absolute uncertainty Δx has the same dimensions as the quantity x itself. Note the measured value of x is *not* just the best value of x. The measured value is the range of values defined by $\pm \Delta x$.

The second form of expressing an uncertainty is the relative uncertainty (f_x):

$$f_x = \frac{\Delta x}{|x_{best}|} \tag{2}$$

Note the relative uncertainty is *dimensionless*. Although, it is convenient to express the relative uncertainty as a *percent* uncertainty. So if $f_x = 0.02$, the percent uncertainty would be 2%. The relative uncertainty is useful when addressing the size of the uncertainty. If $|x_{best}| > \Delta x$, then the relative uncertainty is a number less than 1 or less than 100%. From the example of the length of the arrow we can express our measurement as:

$$l_{arrow} = 5.5cm \pm 2\% \tag{3}$$

The Uncertainty in Repeated Measurements An important test of a measurement's reliability is how well a measurement can be repeated with the same result. The word "same" does not mean "identical" in the context of measurements and uncertainties. Even if a person were to make the same measurement, using the same equipment and the same procedure, the results of the measurements from one trial to another may differ. A good example of repeated measurements is trying to measure the height of students in a class. When you start measuring each student's height you notice that some are taller than others. Yet, you still want a single number that represents height for the students in the class.

So how do you get one measurement and one uncertainty from a series of repeated measurements? For a series of repeated measurements, you can use statistics to describe your data. Your single measurement then is the mean of all of the trials, $\overline{T}$, and your uncertainty is the standard error ΔT. To calculate the mean use the following:

$$\overline{T} = \frac{1}{N} \sum_{i=1}^{N} T_i \tag{4}$$

Here each trial data point is T_i, where i is the ith trial, and N is the number of trials. For example, the value for trial 1 would be T_1. Also, the "average" described here is the mean. Other useful averages include the median (the middle number in an ordered list of values) and the mode (the value that occurs most often). In this class we will use the mean. However, be careful to always ask when someone says average if they are talking about the mean, median, or mode. These values can be very different. To determine the uncertainty for a set of repeated measurements, we first calculate the standard deviation, σ_T:

$$\sigma_T = \sqrt{\frac{\sum_{i=1}^{N} (T_i - \overline{T})^2}{N - 1}} \tag{5}$$

The standard deviation is the root mean square deviation of the T_i's from the average value. Next, we can find the uncertainty in the average value, ΔT, by using the following equation:

$$\Delta T = \frac{\sigma_T}{\sqrt{N}} \tag{6}$$

This is also called the standard error. Fortunately the average and standard error are very common statistical operations. Many programs already have these operations built in to their code. To calculate the average and standard deviation in Excel, enter your measurements in a column. The built in function to average the values is `AVERAGE` and the standard deviation is `STDEV`. To find the standard error just divide the standard deviation by the square root of the number of values.

Here I would also like to bring up the concept of the p-value. A p-value is an important statistic that tells scientists whether or not a set of repeated measurements is related to another set of

repeated measurements. There are whole books on how to calculate p-values correctly. Here we will just focus on one type of p-value calculation that is suited for our purposes in this lab. We will assume: *i*) that the two sets of repeated measurements are independent, *ii*) that the two sets each possibly contain a different number of trials or different standard deviations, *iii*) that we are testing to see if the two sets are the same (that the null hypothesis is true), *iv*) that the two data sets have random errors such that one data set isn't thought to have a higher mean than another, and *v*) that the number of values in each data set is not large (<100). For our purposes, we will assume that if the p-value is less than 0.05 then the two data sets are statistically significant, meaning that the null hypothesis is rejected and the two data sets are different. Any p-value above 0.05 and we will assume that we cannot reject the null hypothesis or we cannot reject the fact that the two data sets may be the same. Remember a p-value above 0.05 does not prove the null hypothesis or that the two data sets are the same. To calculate a p-value use the Excel function `TTEST`. Enter your two data sets as columns in Excel, these columns will be the first two inputs to the function. The third input is a 2 for a two-tailed t-test, since one data set isn't thought to have a higher mean than the other. The fourth input is a 3 because the data sets have variable amounts of data and standard deviations.

1.1.2 Propagation of Uncertainties

Often the physical quantity of interest is not one that can be measured directly but is calculated from other measured quantities. Since all measured quantities have associated uncertainties, how do those uncertainties affect the calculated quantity? Luckily, there are rules for propagating the measured uncertainties into the calculated uncertainties. The most general rule is that for any function g, where g is a function with independent variables $(A, B, C, ...)$ and uncertainties $(\Delta A, \Delta B, \Delta C, ...)$, then the uncertainty of g, Δg, is:

$$\Delta g^2 = |\frac{\partial g}{\partial A}|^2 \Delta A^2 + |\frac{\partial g}{\partial B}|^2 \Delta B^2 + |\frac{\partial g}{\partial C}|^2 \Delta C^2 + ... \tag{7}$$

For example, let's say d is a function of the independent variables l and w, such that $d = l - w$. Then the uncertainty in d, Δd is:

$$\begin{aligned} \Delta d^2 &= |\frac{\partial d}{\partial l}|^2 \Delta l^2 + |\frac{\partial g}{\partial w}|^2 \Delta w^2 \\ &= |1|^2 \Delta l^2 + |-1|^2 \Delta w^2 \\ &= \Delta l^2 + \Delta w^2 \\ \Delta d &= \sqrt{\Delta l^2 + \Delta w^2} \end{aligned} \tag{8}$$

Note that uncertainties of l and w are *added* together. The reason is the measurements of l and w are independent of each other. If they were not independent then we could cleverly choose the uncertainties of l and w that would cancel each other so that d would be an exact value. But since there is no exact value based on a measurement we have to add the uncertainties.

Let's do one more example where we use division. Let's say R is a function of the independent variables V and I, such that $R = V/I$. Then the uncertainty in R, ΔR, is:

$$
\begin{aligned}
\Delta R^2 &= |\frac{\partial R}{\partial V}|^2 \Delta V^2 + |\frac{\partial R}{\partial I}|^2 \Delta I^2 \\
&= |\frac{1}{I}|^2 \Delta V^2 + |\frac{V}{I^2}|^2 \Delta I^2 \\
&= \frac{V^2}{I^2}(\frac{1}{V^2}\Delta V^2 + \frac{1}{I^2}\Delta I^2) \\
\Delta R &= |R|\sqrt{(\frac{\Delta V}{V})^2 + (\frac{\Delta I}{I})^2}
\end{aligned}
\tag{9}
$$

1.1.3 Curve Fitting and Extraction of Fit Parameters

Throughout the semester you will be studying various physical systems and comparing relationships between measured quantities to theoretical predictions. Usually the way to compare experimental data to theory is by fitting a curve. Once you have a theoretical curve fit to the data you can then extract the fit parameters that best describe your data. Probably the simplest relationship to fit to a series of data points, (x, y), is a straight line. It takes two fit parameters (the slope, m, and the y-intercept, y_0) to define a straight line:

$$
y = y_0 + mx
\tag{10}
$$

Once you have fit the line you can then extract the fit parameters, the slope and the y-intercept, which should match your theoretical predictions. Other common fits include polynomials, exponentials, inverse-squares, and sine waves.

1.2 Experiment 1: Measurements of a Block

Now let us apply these techniques for measuring and calculating uncertainties to an actual object. First, using the paper ruler provided, measure the dimensions of a small aluminum block. For consistency we will define the the dimensions of the block length (l), width (w) and thickness (t) as illustrated in Fig. 2. Note that $l > w > t$. Each lab partner should make his or her own measurement and for now keep your measurements to yourself. Do not share your measurements with your partner! Record your measurements in your lab notebook and make sure to list your absolute uncertainties for each dimension. In addition, record your measurements of the block's dimensions with the relative uncertainties as well. Now, record your partner's values.

Q: Do you and your partners answers agree? That is, are your measurements the same to within the uncertainties listed? Why or why not?

Second, let's calculate the perimeter around the face of the aluminum block. Both the length and width of the block have uncertainties. So in order to find the uncertainty in the perimeter (a
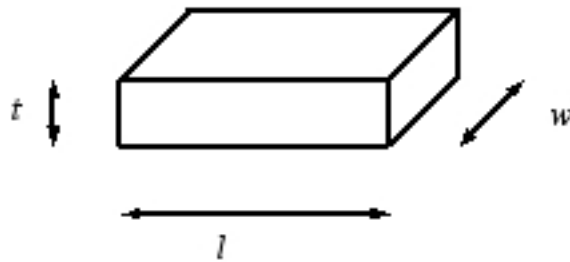
Figure 2: Dimensions of aluminum block

quantity that depends on both values) we need to propagate the uncertainty using the equations from the Introduction. When calculating the values below be sure to use your measured values and not your partner's.

Q: What is the perimeter (p) of the block? Give both the equation and value.
Q: What is the uncertainty in the perimeter (Δp)? Give both the equation and value.

Now, let's calculate another quantity, the surface area (A) of the block.

Q: What is the surface area (A) of the block? Give both the equation and value.
Q: What is the uncertainty in the surface area (ΔA)? Give both the equation and value.

At this point lets make an approximation. If the relative uncertainties $\frac{\Delta l}{l}$ and $\frac{\Delta w}{w}$ are small (close to 0, like 0.01), then the product of the relative uncertainties $\frac{\Delta l}{l}\frac{\Delta w}{w}$ is even smaller and can be neglected.

Q: What is the new uncertainty in the surface area (ΔA)? Give both the equation and value.

Finally, let's use the rules for making digital measurements and propagating uncertainties to determine the density $\rho = m/V$ of the block. First, we need to measure the mass, m, of the block. List this in your lab notebook along with the associated uncertainty. Next, calculate the volume, $V = lwt$, of the block.

Q: What is the relative and absolute uncertainty of the volume? Use the uncertainty of V and m to find the relative and absolute uncertainty of ρ.
Q: How do you improve the uncertainty associated with measuring a quantity with either an analog or digital scale?

1.3   Experiment 2: The Period of a Pendulum

In this experiment we will use the measurement and uncertainty techniques for repeated measurements to determine the period of a pendulum. Use a stopwatch to measure the period of a pendulum. Decide when to start and stop the stopwatch. Each partner should make a series of his/her own measurements; about 5 each. There is a good chance that some of the measurements are different. This is because each time you start and stop the watch you may judge the period a little too long or a little too short from the true period. To determine the best value of the period, calculate the average of your 5 trials. To determine the uncertainty of the period, calculate the standard deviation. Write down both your average and standard deviation and your partner's average and standard deviation.

Q: Do your measurements and your partner's measurements agree to within the uncertainty listed? Calculate the p-value for the data to support your answer.
Q: How can you reduce the uncertainty for a series of repeated measurements?

1.4   Experiment 3: 1D Motion of a Rolling Ball

In this experiment we want to demonstrate how to fit a theoretical curve and extract the various parameters. The example here will be a ball rolling on a smooth level surface. If we assume that there are no net forces acting on the ball, then we can predict that the position of the ball will be *linearly* proportional to time:

$$x(t) = x_0 + vt \tag{11}$$

where x_0 is the initial position (intercept) of the ball at $t = 0$ and v is the velocity (slope) of the ball. To prove that the motion of a rolling ball is linear we want to measure the motion of a rolling ball and fit various curves to our data to see the best fit. First, let's measure the position of a rolling ball using a motion sensor. Follow the instructions in Appendix A to set up the motion sensor. Take a short run of data ($\sim$ 3 sec) of the ball rolling away from the sensor. Plot the data in Excel using a scatter plot. Right click on the data and "Add Trendline". This will pull up a menu where you can fit different curves to your data. Fit both an exponential and a line, making sure to plot the equation on the graph along with the R^2 value. List the initial position, velocity, and R^2 values here and paste the corresponding graph into your notebook. You should also use the Excel `Data Analysis > Regression` function to fit the data to a line as well. The fit will be the same, but this method will show you the standard errors associated with each parameter. If you do not have a `Data Analysis` box under the `Data` tab then you will have to install the Analysis Toolkit Add-in to perform this function.

2 Kinematics: The Bouncing Ball (Formal Report)

2.1 Introduction

The experiment for this week could not be simpler, at least conceptually. You'll have a superball and will be expected to analyze a part of its motion after it is dropped from a height of about 80 cm. Because the experiment itself is so modest, we've decided to use it to give you a taste of automated data acquisition and also an opportunity to use a computer for data analysis. In the actual experiment, you'll use a motion sensor, connected to a laptop computer. Once triggered, the sensor sends out a specified number of pulses (e.g. 100) of high frequency sound with a specified time interval between pulses (e.g. 0.05 seconds). (For the examples given, the pulses are emitted over a time period of 5 seconds). See Fig. 3 for a graph of the emitted signal.
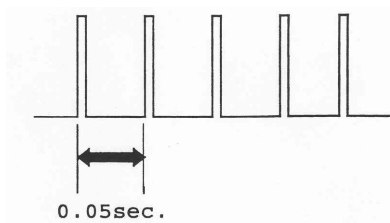
Figure 3: Pulses from the motion sensor

The pulses of sound travel at a speed of about 330 m/s and, in a carefully executed experiment, will be reflected back to the motion sensor by an object along the path of the pulses. If the object is <u>not too far away</u>, and is <u>not moving too rapidly</u>, a given pulse reflected by the object will arrive back at the sensor well before the next pulse is sent out. The motion sensor (in conjunction with the computer) measures the time interval between the emission of a given pulse and the reception of its reflection. From this time difference and the known speed of sound, it calculates the position of the moving object. The sensor-computer assembly can also calculate the time at which a given pulse was reflected from the moving object. The values of time and object position (relative to the sensor) are stored on the computer. They can be displayed on the computer and also transmitted between software applications. The motion sensors work most reliably between about 0.5 m and 1.5 m.

2.2 Experiment 1: Qualitative Analysis of the Bouncing Ball

Without using the motion sensor, drop the ball from a height of about 80 cm above the floor and observe its subsequent motion. Try to keep the motion more or less one dimensional for several bounces. Make a qualitative position-time graph of the motion in your lab book, assuming the motion to be one dimensional. Be sure to indicate whether you have chosen up or down as the positive position direction. Label all axes with variables and units. Also title the graph. Make sure to take some time with this step and carefully observe the ball's motion.

Q: Which points on the graph correspond to the ball's collision with the floor?

Q: Which points correspond to the top of a bounce?

2.3   Experiment 2: Quantitative Analysis of the Bouncing Ball

2.3.1   Data Aquisition and Analysis using Data Studio

1. *Practice your technique before recording data*

   Follow the instructions in Appendix A to set up the motion sensor. Remember that the motion sensor works best when the object is between 0.5-1.5 meters away. Center the ball directly under the motion sensor at a height of about 0.8 m above the floor and at least 0.2 m below the motion sensor (ideally 0.5 m below). Lightly hold the ball and then release it from rest. Practice this several times until the ball initially hits a point near the center of the floor tile directly beneath the sensor and then bounces at least 2 or 3 times within the area of the tile. After releasing the ball you should quickly move your hands away.

2. *Acquire data*

   Once you have a technique for releasing the ball, you are ready to acquire data. Using the motion sensor and Data Studio, record the position of the ball every 0.05 seconds (20 Hz sample rate) over an interval of 5 seconds. This should incorporate a few bounces of the ball. See Appendix A for instructions on the use of the motion sensor and Data Studio.

3. *Check your data*

   Examine a graph of the position vs. time data on the computer. If the ball has not had several bounces within the bounds of the tile, then the display will be very noisy. In that case, repeat your experiment until you get clean data. Once you have recorded satisfactory data, export the position vs. time data to a txt file (see Appendix A for details). Now use Data Studio to take a velocity vs. time trace. Print the graphs out of Data Studio and put them into your lab notebook.

4. *Think about your data*

   Answer the questions below for the part of your graph that contains two or more "good" bounces.

   Q: How does the display compare with your sketch of the results from the preliminary experiment? Comment on any differences.

2.3.2   Data Analysis using a Spreadsheet Program like Excel

1. Open the data of time (t) and position (y) in a blank spreadsheet in Excel (see Appendix A for details). Be sure to label your columns and indicate in the column headings the units.

2. Make a graph of position vs. time. Note the approximate times corresponding to one good bounce (one parabola).

3. Next, we want to make a velocity vs. time graph. To do this we will have to perform some calculations on our position and time data. In an empty column next to the data, calculate the mean time between two consecutive data points. Do this for each of the times recorded, using the time point and the one below it. The last data point will not have a "mean time between points" data point. To quickly obtain the mean time for the whole column you can use a formula. For example two consecutive times may be stored in cells `A3` and `A4`. Then the formula for the mean time would be `=(A4+A3)/2`. Copy and paste this formula to the rest of the column. Now, in the next empty column calculate the mean velocity between consecutive data points, using the data point and the one below it. Again the last data point will not have a value. Remember that the mean velocity between two points, say point 1 and point 0, is $(x_1 - x_0)/(t_1 - t_0)$. Thus, if your two consecutive times are stored in cells `A3` and `A4` and two consecutive positions are in cells `B3` and `B4` then the formula for the mean velocity between points would be `=(B4-B3)/(A4-A3)`. Fill the other relevant cells in the column with this formula, as well. Now that you have calculated the data you should be able to make a graph of mean velocity between consecutive data points vs. mean time between consecutive data points.

   Q: Why must we plot the mean velocity between consecutive points against the *mean* time between consecutive points, as opposed to just the time trace?
   Q: Do you think it is reasonable to regard this graph as a graph of instantaneous velocity versus time? Explain your answer briefly.
   Q: How does your velocity vs. time graph that you made in Excel compare to the one in Data Studio? If there are any discrepancies explain them.
   Q: What can you say about the acceleration of your bouncing ball? Calculate the mean acceleration between consecutive points using the point and the one below it to support your answer.

4. Locate the first and last points on the best-looking bounce; that is to say, isolate the best bounce from the top of the arc of the bounce to the point where the ball meets the floor. This is usually easier to judge from the velocity vs. time graph than from the position vs. time graph. Generate a new graph of mean velocity between consecutive points vs. mean time between consecutive points from a subset of points that represent this one bounce. Use Excel `Data Analysis > Regression` to obtain the slope and intercept of the best-fit straight line that can be passed through the subset of data points. Obtain this slope and intercept for the best fit straight line passing through the graph you have produced. If you are having trouble obtaining the slope and intercept refer to the directions from the last lab.

   Q: What value for the slope do you get, and what is its statistical uncertainty? What is the physical significance of the slope?

5. Label the columns of your spreadsheet (including units), and the axes of your x vs t and v vs t graphs, if you've not already done so. Print out the graphs.

6. Write a summary of your results in your lab notebook. Make certain you have answered all the questions. Include your printouts (staple, tape or glue).

2.4   Formal Lab Report

The report for this lab is to be a formal report giving a complete description of your lab work. You should assume that the reader has a physics background equivalent to this course, but knows nothing about what you did in the lab. Your report should be sufficiently complete so that the reader will know exactly what you did and can understand the significance of your results. The format for a formal lab report is detailed in Appendix D.

3 Force: Acceleration on an Inclined Plane

It is very important to have a thorough knowledge of this procedure before beginning the lab. You will run this experiment twice, for two inclination angles. Also, review Appendix E (Experimental Uncertainty Analysis).

One of Galileo's great contributions to experimental science was his use of the inclined plane as a means of "diluting" gravity; that is, as a way to slow down free fall so that precise measurements of the motion could be made. A modern refinement of Galileo's rolling-ball inclined plane is the air track. On the air track, gliders are supported by a thin layer of air and move nearly frictionlessly along the track.

Your job is to repeat Galileo's inclined plane experiment of measuring $g = 9.80$ m/s² to an accuracy of $\pm 1\%$. [You may not be able to achieve this accuracy, but this is what you will try for. A very rough rule of thumb in physics is that a 10% measurement of something is fairly easy; a 1% measurement requires considerable thought and care, and a 0.1% measurement is apt to be extremely difficult.]

3.1 Introduction

For this lab we will assume the following:

1. Here we will assume that frictional effects are completely negligible, so a cart should move down a tilted airtrack with *constant* acceleration, a. [Frictional effects include not only possible friction between the glider and the track, but also drag due to air resistance.]

2. Also, if we assume that there is no friction, then the numerical value of a would be given by

$$a = g \sin \theta = g \frac{h}{L}, \tag{12}$$

    where $g = 9.80$ m s⁻², and L and h are simply geometrical quantities in a right triangle describing the tilt of the track as shown in Fig. 4. The point here is that if we measure a, h, and L; then we can then compute g from Eq. 12.

3. We will assume that the kinematic relations for the case of constant acceleration are applicable to the motion of the cart on the airtrack. In particular, the coordinate position s of the cart as a function of time, t, on the track is described by

$$s = s_0 + v_0 t + \frac{1}{2} a t^2. \tag{13}$$

where s_0 is the position at $t = 0$ and v_0 is the velocity at $t = 0$.

Important Note: It is essential that you be prepared to calculate quickly the experimental values of g from your observations, so that you can see how the results are coming out and be ready
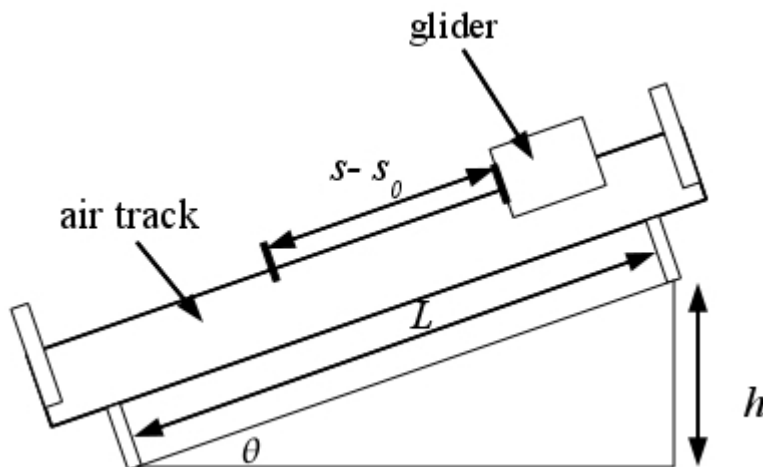
Figure 4: Definition of the parameters involved in the inclined plane. Note that L and the maximum value of s are not necessarily the same.

to modify procedures if necessary. [It is always a good idea to do preliminary analyses of at least some of the data while in the lab, if possible. It may turn out that either you have forgotten to take one vital measurement, or that the results are very peculiar and should be repeated, perhaps with a change in technique. This is just as true in a "real" experiment as in an introductory course. For example, a clinical team might decide to run a drug trial on 1000 individuals that will take 1 year. If the team waits until the end of the year to analyze the data, they may not know for a year that they need a second control group. However, if they had done a preliminary analysis of 50 patients at the two week mark, they could have quickly changed their initial experimental parameters and saved a year!]

3.2 Experiment 1: Measuring g using a Stop Watch

1. *Level the track and determine the uncertainty*

   Level the track by adjusting one of the leveling screws. Adjust only the screw at the end where there is a single one; if you fiddle with the two at the other end, you will change the side-to-side leveling which has already been adjusted. You will want to know how accurately you can level the track. Here's one way of doing that: with the track "level," put a file card under the "foot" at one end. Does that definitely make it *not* level? Or do you need 2 or 3 or 4 cards? Now remove the cards and find out how thick a file card is. Use a Vernier Caliper for this measurement. These tests should give you a good idea of the uncertainty in h.

2. *Measurement of a*

Put an aluminum riser block of your choice under the foot of the air track. Release the glider from rest and time (using a stopwatch) how long it takes to travel a chosen distance $s - s_0$. Use Eq. 13 to calculate a. Repeat 4 more times. Find the mean of the your five trials to get the best value of a. The uncertainty on this value is given by the standard error. (For how to calculate these values and other uncertainties refer to Appendix E.) Make sure each partner takes their own data.

3. *Best estimate of g*

   Now using your best values of h, L, a and Eq. 12, calculate the value for g. This is your best estimate. Use the uncertainties of h, L, a and error propagation to determine the uncertainty of g. The agreement with the standard value for g ought to be reasonably good. If you are off by 5% or more, you probably made a mistake somewhere in your measurements or calculations. Check them over.

   Q: What is the precision of your estimate? What is the accuracy of your estimate? (For this question you will need to know the definitions of precision vs. accuracy before you come to the lab.)

4. *Correcting g*

   There are two main reasons why your preliminary measurement of g may be inaccurate: **timing accuracy** and **air drag**. The first reason is that your timing accuracy may be poor. It is clearly vital that the stopwatch be started and stopped "correctly." If the watch is to be started upon release of the cart, and stopped when it hits the bottom of the track, these two events are qualitatively quite different: the start is rather slow, while the stop is quite fast. Thus, "reaction time" errors may not cancel out. The second reason is that air drag may be significant and we assumed that frictional effects would be small. For a cart going *downhill*, air drag would tend to slow the cart down and to make the apparent value of g *smaller* than expected. To correct for this, start with the cart traveling *uphill*, and analyze a complete trip up and back down the track. Calculate your new g value; is it the same, larger, or smaller than expected? Why?

   Q: How could you limit the impact of these two errors on your measurement?

3.3 Experiment 2: Measuring g using a Motion Sensor

1. For your new experiment, initially use a $1''$ riser block.

2. *Automate the timing technique*

   Use the motion sensor instead of a stopwatch. The motion sensor should be set to take 100 readings at 0.2 sec intervals and situated ~ 160 cm along the airtrack, with its height set so that its center is $\sim 7\frac{1}{2}$ cm above the track.

   > **CAUTION: make certain that the cart on the airtrack never collides with the motion sensor!**

3. *Release the glider and record data*

   Decide from where your glider will start and during which part of its motion you will record data. Then record the data using Data Studio. Have each partner record data.

4. *Check your data quality*

   If you get a good quality position vs. time graph, you're ready to analyze the data. If your position-time graph is "noisy", try again. If you can't eliminate the noise, see one of the instructors. See Appendix A for data transfer instructions.

5. *Analyze your data using a spreadsheet*

   Once you have a good graph, analyze part of your data as you did in the Bouncing Ball lab. That is select a good section of position vs. time data. Then create a velocity vs. time graph by calculating the mean velocity between consecutive points and the mean time between consecutive points. Fit a line to the velocity vs. time data and extract the slope, this is g. Also, obtain a value for the *statistical* uncertainty in g by performing a linear regression and looking at the statistics.

6. *Calculate the experimental uncertainty*

   Consider all sources of uncertainty in each of the measured quantities that go into the determination of g (see Appendix E in this lab manual) to estimate an experimental uncertainty in g that is due to the apparatus and measurement technique.

7. *Repeat with a different track angle*

   Repeat your experiment with the motion sensor using a different riser block, again acquiring one data set for each partner. You may need to change the motion detector time interval for this set of measurements. Does your result for g improve? Why or why not?

4 Momentum: Elastic and Inelastic Collisions

Read the instructions completely and carefully before coming to lab. Also, work out how to calculate the uncertainty in the momentum ratios ($\frac{p_f}{p_i}$). You will need to work efficiently in this lab otherwise you may end up staying quite late.

4.1 Introduction

In this experiment we will use the conservation of linear momentum to study two types of collisions: perfectly **inelastic** collisions where the two colliding bodies adhere to each other upon contact and move off as a unit after the collision, and perfectly **elastic** collisions, where the two colliding bodies bounce off one another upon contact and move as two separate entities after the collision. Linear momentum p for an object is equal to mv, where m is the mass of the object and v is the object's velocity. This means that a heavier or faster object will have more momentum than a lighter or slower object. Since linear momentum is conserved, if objects collide, they can change their momenta, but the overall sums of the momenta have to be equivalent. This is given by the equation:

$$\sum_{i=1}^{N} p_{\text{i,initial}} = \sum_{i=1}^{N} p_{\text{i,final}} \tag{14}$$

For two objects colliding in a perfectly elastic collision the equation becomes:

$$m_1 v_{1,initial} + m_2 v_{2,initial} = m_1 v_{1,final} + m_2 v_{2,final} \tag{15}$$

and for two objects colliding in a perfectly inelastic collision the equation becomes:

$$m_1 v_{1,initial} + m_2 v_{2,initial} = (m_1 + m_2) v_{final} \tag{16}$$

4.2 Experiment: Elastic and Inelastic Collisions

1. *Prepare the equipment: level the airtrack and test PASCO Motion Sensors*

   Start by leveling your air track. Setup *two* PASCO motion sensors positioned ∼140 cm apart, *facing* each other. The sensors should be centered over the airtrack. The sensors should be ∼6 cm above the track and pointed at the gliders below.

   > **Do not allow the gliders to crash into the sensors. The sensors can easily be damaged. Make sure the gliders move freely under the sensors.**

2. *Determine the masses of the gliders*

   The air-track gliders have <u>approximately</u> the following masses:

small glider 200 grams
large glider 400 grams

You should check the masses of the carts you use on the electronic balance. List their masses and uncertainties here.

3. *Test the motion sensors with a single glider*

Push one glider along the length of the airtrack and record its position using both motion sensors. Remember, the sensors measure distances relative to themselves. Look at the graph of position vs. time. Based on your qualitative inspection of the graph, what can you say about the velocity of the single glider? The two plots from the two sensors will look very different because one sensor will measure the cart as it comes closer to the sensor and the other will measure the cart as it gets farther away from the two sensors. How do you reconcile the two plots from the two sensors? Print out your plots and paste them into your lab notebook.

4. *Measure the momentum transfer during a two body collision*

Here we are going to measure the positions of two gliders colliding with each other and verify that momentum is conserved. Place the small elastic glider at rest, between the sensors. Push the second small elastic glider into the first. Look at the graph of position vs. time for the two gliders.

Q: Can you identify the moment of collision?

Now use Data Studio to look at the velocity vs. time graph for the collision of the two gliders. Keep in mind that one of the sensors is looking *backwards*.

Q: How does this affect the velocity it measures? How will you correct the velocity?

Extract the initial and final velocities for each glider. List the velocities along with their uncertainties in your lab notebook. Also, print out your plots and paste them into your lab notebook.

To verify that momentum is conserved, first calculate the total initial momentum of the two gliders before impact, p_i. From the equations above this should be $\sum_{i=1}^{N} p_{i,\text{initial}}$ or in the case of an elastic collision $m_1 v_{1,initial} + m_2 v_{2,initial}$. Second, calculate the final momentum of the gliders after impact, p_f. Now calculate the momentum ratio, $\frac{p_f}{p_i}$. Finally, use error propagation to calculate the uncertainty in the momentum ratio.

5. *Repeat the two body collision measurement for 3 cases of collisions*

For both elastic and inelastic collisions, you will investigate at least three cases (not necessarily in the order given below). The minimum three cases are:

   (a) small or large cart into another stationary cart of equal mass
   (b) small cart into large cart at rest
   (c) small or large cart into moving cart (equal or unequal mass)

This will give you a total of 6 data sets. You have already completed the data set for an elastic small cart bumping into an elastic small cart at rest. This leaves 5 data sets left to do. For each data set record the position vs. time data for the collision. Print the graphs and paste them into your lab notebook.

4.3   Analysis

For the 3 cases and 2 kinds of collisions (elastic and inelastic) you will have a total of 6 data sets. For each case, you should also extract the initial and final velocities and their uncertainties for each cart. You can do this is one of two ways:

- Position vs. Time: for each data set there are 4 distinct line segments which represent the 4 velocities of the gliders (2 before collision velocities and 2 after collision velocities). If the line segments are in fact lines, what can you say about the velocities? Use regression analysis to find the velocities of the gliders before and after the collision and the uncertainty.

- Velocity vs. Time: use the position vs. time data to calculate the velocity vs. time data as in the Bouncing Ball lab. Make graphs of velocity vs. time and identify the velocities of the two gliders before and after the collision. Make sure to calculate an average velocity for each of the 4 velocity segments, the standard error will be the uncertainty.

Finally, calculate the final linear momentum (magnitude)/initial linear momentum (magnitude) (p_f/p_i) and its uncertainty for each case. Remember that the final momentum equation will be depend on the type of collision: elastic or inelastic. Also, you must work efficiently if you don't want to be here too late! Make sure to compile in your lab notebook a table of the ratios that you calculate, along with their uncertainties.

Q: What is the expected ratio if momentum is conserved? Do your ratios agree with this expected ratio? Why or why not?

5 Conservation Laws: The Ballistic Pendulum (Formal Lab Report)

Make sure to read and understand this handout in its entirety before coming to and beginning the lab. Work out the derivations needed to complete the calculations in §5.2.

5.1 Introduction

The ballistic pendulum (see Fig. 5 below) was, once upon a time (before the era of high-speed electronics), used to measure the speeds of bullets. The method used to determine the speed of the bullet depends on the clever use of fundamental conservation laws in physics.
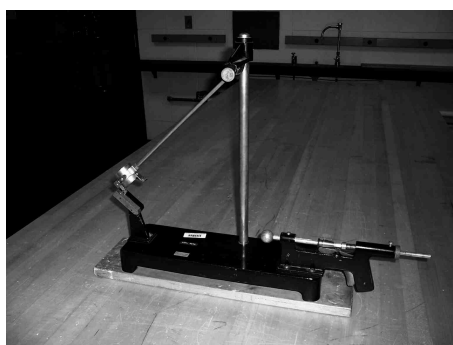
Figure 5: Ballistic Pendulum

A bullet, with mass m and initial speed v, collides with and becomes lodged in a mass M. (See Fig. 6.) The larger mass M is supported as a pendulum. The collision itself takes place so rapidly that m and M together can be considered as a closed system during the collision, i.e. as two particles which interact only with each other.

Immediately after the collision, the new composite object $(m + M)$ has a velocity to the right, v'. Then $(m + M)$ acts as a pendulum and swings upward, trading kinetic energy for gravitational potential energy. The amount of potential energy gained is given by $(m + M)gh$, where h is the height through which the center of mass of $(m + M)$ rises. In this lab you will calculate the muzzle velocity v (or, more properly, the muzzle speed) of the bullet using a set of measurements of h. Using this information, you will then predict the range of the bullet (if it were not intercepted by the ballistic pendulum) and you will dramatically test your prediction. Before class, you should

- derive an expression for v', the speed of the composite object just after the collision, in terms of h and the masses. This can be done by considering the conservation laws which apply *after* the collision.

- relate v' to the bullet's muzzle velocity v by considering the conservation laws which apply *during* the collision. Together with the previous expression, this gives an equation for v in terms of h.

- *predict* how far the bullet would travel if it didn't hit the pendulum and actually continued unimpeded until it struck the floor a distance H below.
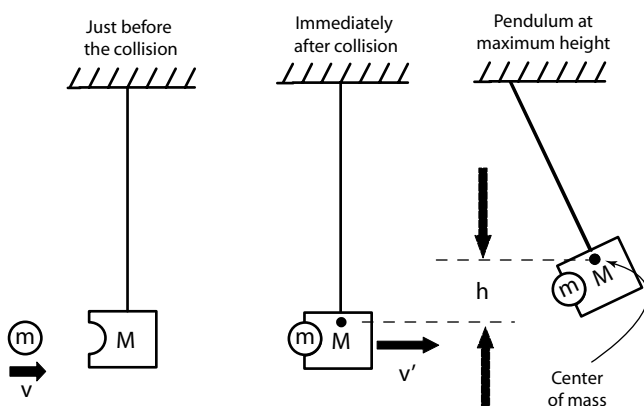
Figure 6: Ballistic Pendulum before and after collision

5.2   Experiment: The Ballistic Pendulum

Predict the horizontal distance, H, the bullet will travel before striking the floor. Start by doing the derivation outlined below using only variables (before coming to lab), then measure values for the various parameters. Once you have a predicted range and its uncertainty, test your prediction.

1. *Measure h and its uncertainty*

   Be careful to measure h properly. For a point mass, the change in the gravitational potential energy is $\Delta U = mgh$. For an extended object, this same expression holds if h is the change in vertical position of the **center of mass**.

   There is a set of notches to catch the pendulum at approximately its highest point. The notches are numbered but the numbers are simply labels. You must measure the appropriate value of h directly with a caliper.

   You will want to make <u>a series of measurements of h</u> (at least 7) in order to determine its uncertainty.

2. *Weigh the projectile (m) and the pendulum bob + hanger (M)*

   The bullet can be weighed. Make sure you keep track of your bullet as its mass (m) is important and the bullets don't all have the same mass.

The larger mass M can also be removed from the pendulum for weighing.

3. *Calculate the horizontal range, H, of* **your** *projectile*

   Don't forget to calculate the uncertainty!

4. *Mark your prediction*

   Tape paper and carbon paper to the floor so that the bullet makes a distinct mark. Mark the expected impact point <u>and the uncertainty range</u>.

5. *Test your prediction*

   Be careful! The bullets move at fairly high speeds.

   Summon one or more of the instructors and let us see how good your prediction is. When testing your prediction, do not be confused by bounces!

   Carry out a series of trials (at least 5) of the horizontal range of the bullet as a projectile. Record the *actual* distance the bullet travels for each trial, and compare with prediction.

5.3   Formal Report

Your write up should focus on how you measured each of the relevant quantities and how you determined the uncertainty for each of them.

- Explain carefully how you used the observed value of h to calculate v and how you used this value to predict the horizontal range.

- Describe how well you did at predicting the horizontal range. (**Uncertainties** for the expected and measured values of the horizontal range should be included.) Do you notice any trends in your data, anything systematic? If so, comment on this.

- Discuss the conservation or non-conservation of kinetic energy in this collision. Was the kinetic energy of $(m + M)$ just after the collision equal to the kinetic energy of the bullet just before the collision? If not, **what percentage of the bullet's initial kinetic energy was lost?**

Read Appendix D for important instructions concerning formal lab reports.

6 Rotation: The "Outward Force" Apparatus

Make sure to read and understand this handout in its entirety before coming to and beginning the lab. You will conduct this experiment twice, for two different angular velocities.

6.1 Introduction

6.1.1 Background on Rotational Motion

Consider a body in uniform circular motion as shown in Fig. 7. The body might be, for example, a ball on the end of a string. We know from kinematics that a body in uniform circular motion is accelerated: although the speed remains constant, the velocity vector is continually changing in direction. We also know that the acceleration is a vector directed toward the center of the circle with a magnitude of

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}, \tag{17}$$

where r is the radius of the circle, and T is the period of the circular motion.
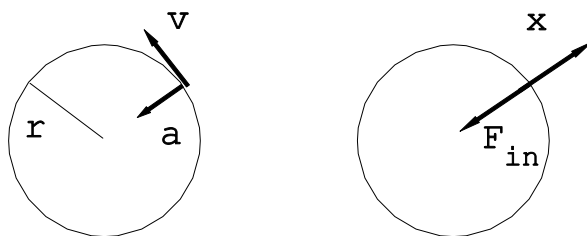
Figure 7: Body in circular motion and its force diagram

From our study of dynamics we know that the acceleration a body experiences can be predicted from a knowledge of the forces acting on it. What are the forces acting on the ball? The string is surely pulling on it; so, there is an inward force of magnitude F_{in} as shown in Fig. 7. Childhood experience also leads us to suppose there is an outwardly directed force, whose magnitude is labelled X in the diagram. But how large is X? In this lab, we will suppose that there is indeed an outward force experienced by a body undergoing uniform circular motion. We will do this as a means of reconciling some perhaps preconceived notions about "centrifugal force" with the existence of actual forces in such a system. Presuming the existence of X, the magnitude of the net inward force would be $F_{\text{net}} = F_{\text{in}} - X$, and we can use Newton's Second Law to write

$$F_{\text{net}} = F_{\text{in}} - X = ma_c \tag{18}$$

and therefore

$$X = F_{\text{in}} - m\frac{4\pi^2 r}{T^2}. \tag{19}$$

In this experiment, instead of using a ball on the end of a string, we will use a cylindrical mass attached to a spring. The only property of the spring we need to know is that, by Hooke's law, the force exerted by the spring (that is, F_{in} in Fig. 7) depends only on how much the spring is stretched. Thus F_{in} can be measured (using the apparatus described below) by a separate experiment in which the spring is stretched by the same amount as under rotating conditions. Since F_{in}, m, r, and T can all be measured, Eq. 19 can be used to determine the experimental value of X. The key to achieving sensible results will be in thoughtfully determining uncertainties in all measured values, as described later in these notes. Make sure to pay special attention to this aspect of the experiment as you go along.

6.1.2   Description of the "Outward Force" Apparatus
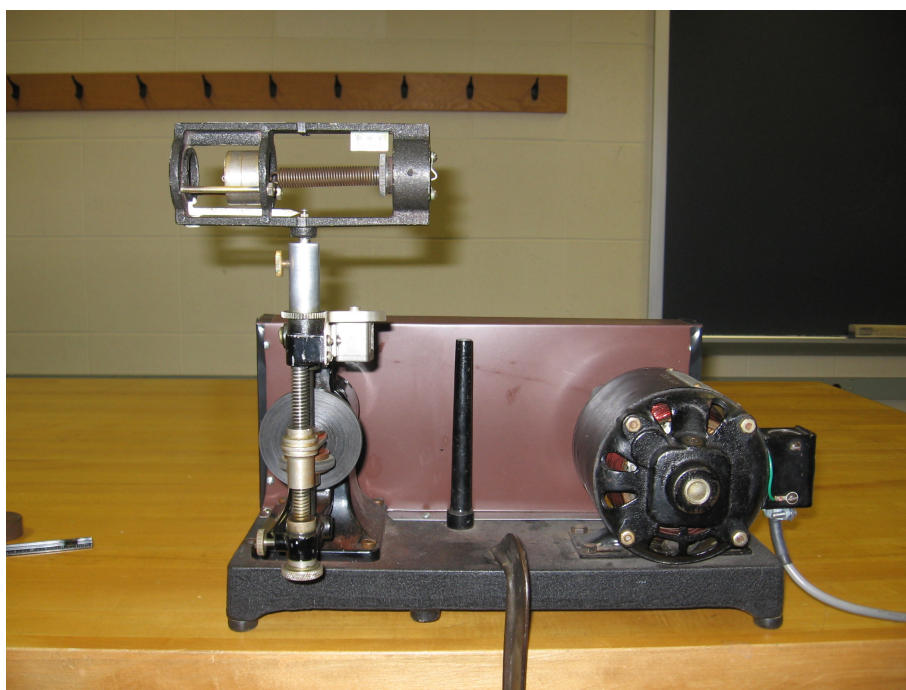
Figure 8: View of entire apparatus

The outward force apparatus (Figure 8) is an experiment that allows you to calculate both the force acting on a rotating mass and the acceleration of the rotating mass independently. This is important because we want to compare F_{in} and ma_{c} so we need independent measurements of force and acceleration.
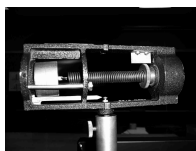
Figure 9: Cross-section view of the carriage attached to the rotation device

The apparatus consists of a carriage, as shown in Figure 9, that holds a mass connected to a spring. The mass is attached to the spring, but is otherwise free to slide back-and-forth in the horizontal direction within the carriage cylinder. The carriage has a small vertical rod that is attached to its base and fits into the cylindrical holder at the bottom of the picture in Figure 9. This cylindrical holder is the rotator shaft. The thumbscrew on the rotator shaft screws in to tighten the vertical rod and secures the carriage to the rotator shaft. The rotator shaft is attached to a rotator device, as shown in Figure 8, that spins the entire carriage at various constant speeds. From the angle of this picture the device would rotate the carriage in and out of the page. This rotation causes the mass to slide outward increasing the distance from the center of rotation and also activating the spring. In the picture, the mass is fully extended to the outermost position. In Figure 10, the mass is in it's innermost position and is still stretched. The rest position of the mass corresponds to the center of the carriage, which also happens to be where the axis of rotation is.
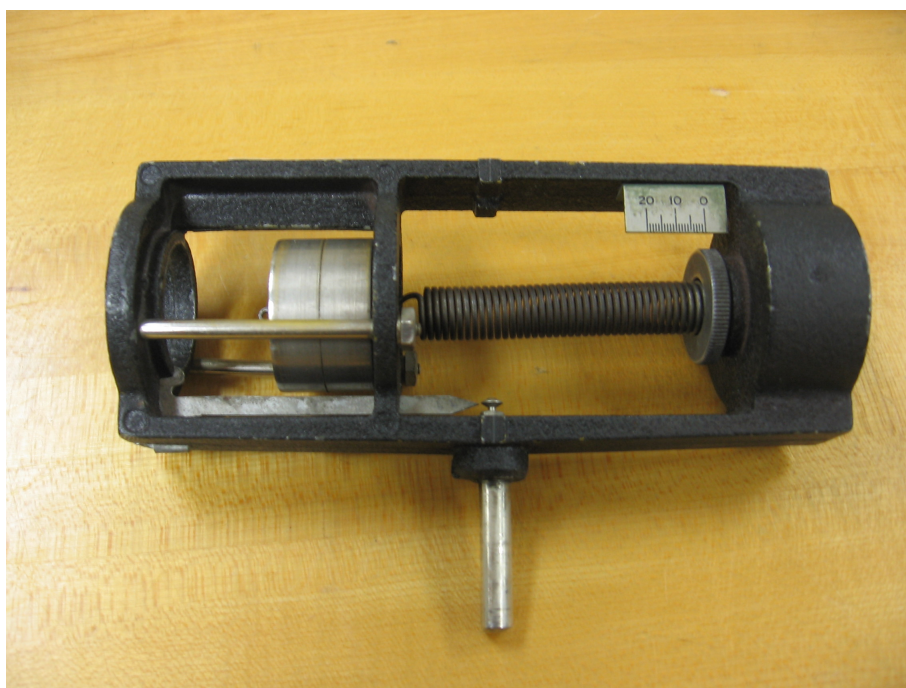
Figure 10: Cross-section view of the carriage

The way the apparatus allows for independent measures of force and acceleration is by measuring acceleration using uniform circular motion described in Eq. 17, and by measuring force using the extension of a spring. First, let's describe how we measure acceleration using the apparatus. For any particular (constant) speed, the period of rotation (T) is constant, and the mass (m) rotates at a constant distance (r) from the center, corresponding to uniform circular motion. You will measure the period of rotation by using a stop watch and a counter that counts the number of rotations. The distance, r, you will measure with a vernier caliper. Using these two values you can calculate the acceleration, a_c, from Eq. 17. Second, to find the force you measure the extension of the spring during rotation, which happens to be r, as well. Then you take the carriage off of the rotation device and attach a weight to the spring to extend it the same distance r. The force due to the weight is then your force, F_{in}. Finally you can compare ma_c and F_{in} to see if they are the same, or if there is a mysterious centrifugal force, X.

6.2 Experiment: Measuring the Mysterious Centrifugal Force of "Murky" Origins

1. *Make a hypothesis about X*

   First, draw a free body diagram of the mass in the carriage. Pay particular attention to the direction of F_{in}. What is the direction of a_c? From this free body diagram do you expect a nonzero X force?

2. *Assemble the apparatus*

   There is a thumbscrew at the far right in Fig. 9 that is attached to the spring in the carriage. Tightening or loosening this spring thumbscrew will change the spring tension. Begin with the spring thumbscrew screwed in as far as possible. Extending from the carriage is an 8 mm diameter cylindrical vertical rod, as described above, with one flat face. Mount the carriage on the rotator device by inserting this rod into the rotator shaft and tightening a locking thumbscrew (this is a different thumbscrew from the spring thumbscrew in the carriage). Be sure that the locking thumbscrew presses against the flat face of the rod.

3. *Adjusting the rotation rate and measuring the period (T) and mass distance (r)*

   The rotation rate can be controlled by the experimenter by an adjustment thumbscrew, which is at the bottom left corner in Figure 8. As you move the thumbscrew the position of a crank shaft on a rotating wheel changes. The velocity is greatest at the edge of the wheel so moving the adjustment thumbscrew such that the crankshaft moves down, toward the edge of the wheel increases the rotation rate. The period T can be measured with a stopwatch and a counter by counting the number of revolutions occurring during a set time period. (Use a fairly large total time in order to minimize the effect of error due to your reaction time in starting and stopping the stopwatch.) The counter is located just below the rotator shaft and the locking thumbscrew. To avoid the problem of trying to determine r while the apparatus is in motion, a pointer device is built into the apparatus. You can adjust the rotation so that

the mass assumes the value of r needed to **just activate** the pointer. The pointer is located in the carriage, on the left side, just below the spring and mass. The point of the pointer is in the center of the carriage, right next to a small cylinder. If the pointer is not activated the pointer points to the bottom of the carriage and cylinder. If the pointer is just activated it points to the top of the cylinder as in Figure 9. If the pointer is overly activated then it points to the top of the carriage above the cylinder.

You will make three measurements of T at three different rotation rates:

   i. Measure the "best" rotation rate, at which the mass barely activates the pointer.

  ii. To get an upper bound, find the *lowest* rotation rate at which the pointer is consistently in the "up" position. Note that rotation rates higher than this will also give a consistent "up" position because the mass becomes pinned against the end of the carriage. The idea is to place your upper bound as close to your "best" value as possible.

 iii. Place a lower bound by finding the *highest* rotation rate at which the pointer is consistently in the "down" position.

4. *Measure r*

    To measure the r where the pointer is just activated, you need to use a fun little screw gadget. With this gold colored screw gadget you can hook your spring to the gadget and by screwing in a screw pull the mass so that the pointer is just activated. Then you can use calipers to measure the distance from the axis of rotation (center of the carriage) to the center of mass. Since this mass is not a point mass, the manufacturer has obligingly marked a line on it to indicate its center.

5. *Calculate a_c and ma_c*

    From the measurements described above, compute the magnitude of the centripetal acceleration, $\frac{4\pi^2 r}{T^2}$, and compare it to the familiar acceleration due to gravity, g, to get a feel for how large a_c is in this particular rotating system.

6. *Determine F_{in}*

    To determine the magnitude of the force F_{in} exerted by the spring when m is in the critical position that actuates the pointer, you can suspend the carriage vertically and attach hanging weights until m is pulled down the appropriate distance. Be sure to determine an uncertainty for this quantity as well.

7. *Repeat the experiment twice*

    Repeat the experiment with the spring thumbscrew at its maximum point away from the carriage side, and again with the screw halfway between maximum and minimum positions.

6.3 Analysis

Make a plot of F_{in} vs. ma_c. Be sure to include error bars that account for the combined uncertainties of all variables used to calculate F_{in} and ma_{c}. Is your data consistent with the equation $F_{\text{in}} = ma_{\text{c}}$, or do you find evidence for a nonzero X force? If there is no outward centrifugal force, X, then why does the mass extend outward when you rotate the carriage?

7 Static Equilibrium: The Force Table and the Moment of Force Apparatus

7.1 Introduction

Static equilibrium is governed by two equations. The first is that the sum of the forces (F) acting on a static object must equal 0, or $\sum_{i=1}^{N} F = 0$. This means that a static object does not translate. The second is that the sum of the torques (τ) acting on a static object must equal 0, or $\sum_{i=1}^{N} \tau = 0$. This means that a static object can't rotate. In this lab we will use both of these principles and vector algebra to measure, calculate, and experiment with objects at rest.

The force table apparatus (Fig. 11) provides a simple means for testing that the sum of the forces on a static object must be zero. The face of the table is a heavy metal disk calibrated around the edge in degrees (360° total). A brass ring, held near the center of the table by a vertical pin, carries three attached strings, each of which can be directed over a pulley fixed to the table edge and terminated with a hanging weight pan. The addition of weights to these pans creates a <u>tension</u> in each string. Each string is thus said to pull on the brass ring and create a force. By using vector algebra we should be able to create positions where the three forces cancel one another and the brass ring remains in the middle of the table.

Figure 11: Force Table

The moment of force apparatus (Fig. 12) allows us to test the second governing equation that the sum of the torques on a static object must be zero. The torque, τ, or moment of force as engineers call it, is caused by a force, F, acting at a distance from the axis of rotation, x, and is given by:

$$\tau = Fx\sin(\theta) \tag{20}$$

where θ is the angle between the force and the distance vector. See Fig. 13. In the moment of force apparatus a torque is created by hanging a weight from a ruler that is balanced on a knife-edge called a fulcrum. Weights can be suspended from the ruler at different positions causing the force to act at different distances from the fulcrum, creating various torques on the ruler. By calculating

the different forces due to the weights and measuring the distance along the ruler we can calculate the individual torques and create arrangements where the ruler is in static equilibrium.

Figure 12: Moment of Force Apparatus from the Cenco catalog
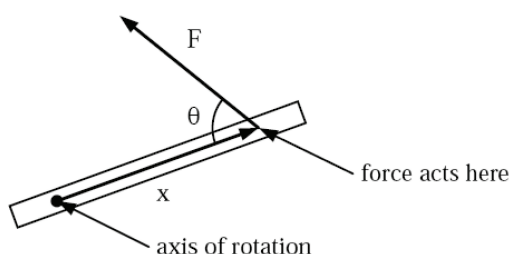
Figure 13: Diagram of the definition of torque

7.2 Experiment 1: The Force Table

Examine the force table example set up in the lab in order to appreciate what your goal will be. Notice that if the weights and string angles are adjusted just right, you can remove the restraining pin, and the ring is held in equilibrium by the action of the three string forces alone. See Fig. 14. However, there is nothing unique about the choices used in the example: there is an infinity of combinations of "string-directions-plus-weights" that will permit equilibrium of the ring. Your task is to show that for other combinations, your chosen values are explained by vector algebra.

7.2.1 Procedure

The procedure consists of two trials:

1. For the first trial, you will experimentally determine a force that moves the ring into static equilibrium.

   (a) Decide on weights and angles for two of the strings. Start with at least 100 grams on each of the two strings. Do not use bare hangers and do not neglect the weight of the hangers. The two positions you select will remain fixed during the rest of the trial. If you wish, you may use "easy" angles like 0° and 90°, or 0° and 120°.

Figure 14: Top view of the force table

(b) Without prior calculation, apply a weight to the third string and vary its magnitude and angle until the third string balances the forces exerted by the first two.

(c) Once you have verified that the balance is "pretty good", estimate uncertainty by adding or subtracting weight until the ring has moved noticeably off-center, or by moving the angle in both directions until the ring is off center.

(d) Finally, verify your experimentally determined force. Using the weights and angles for the first two strings, either numerically or graphically, calculate the third force on the ring.

2. For the second trial, you will calculate the theoretical magnitude and direction of the third force *before* you test to see if your calculations are consistent with experiment.

(a) Choose two new weights set at new angles. <u>Do not</u> separate the first two strings by "easy" angles like 90°. The strings should also be weighted with unequal masses.

(b) • One of the lab partners should conduct a rough graphical analysis (using a ruler and protractor) to calculate the third force with a scale drawing.

• The other partner should conduct a numerical analysis (using trigonometry). To guide your application of vector algebra to this situation, we recommend that you mentally impose an XY-coordinate frame on the table face with the origin at the table center, and one axis aligned with the zero degree marker.

7.2.2   Results

The results should now be compared and any differences reconciled. For each trial list the results in a table, like the one below. Note that in this example case there is not good agreement between the predicted and measured results. Thus, some further investigation is needed. Explain in your lab notebook why there might be any discrepancies.

	Magnitude (N)	Direction (degrees)
Theory (graph)	2.30	120
Theory (numerical)	2.31	119
Experiment	$2.05 \pm .02$	129 ± 2

7.3   Experiment 2: The Moment of Force Apparatus

Your goal in this experiment will be to balance a ruler while various weights exert different torques, just like in the last experiment the goal was to balance the center ring while different weights applied different forces. Again there are many combinations that will allow the ruler to remain in static equilibrium; your job will be to find two ways. Remember that the force being exerted on the meter stick by a weight with mass, m, has a force equal to mg. The distance vector, x, can be determined by the ruler itself. The angle, θ, will always be 90° so that $\tau = mgx$.

7.3.1   Procedure

The procedure consists of two trials:

1. For the first trial, you will experimentally determine the balancing torque.

   (a) First, place the hanger with the 50 cm mark at the center of the fulcrum and check to make sure it is balanced. Why is this the case? Note: some of the rulers may be a little beat up, so the point of balance might be slightly off from the 50 cm mark. If it is >1 cm from the 50 cm mark, please get another ruler, as the uneven mass of the ruler itself will throw off your calculations.

   (b) Second, add a hanger with a weight on it. Do not use bare hangers. This weight will remain fixed during the trial. Write down the mass, position, and torque due to the weight.

   (c) Without prior calculation, apply a weight to the other side of the ruler (with a different mass from the first weight) and vary its magnitude and distance until the ruler balances on the fulcrum.

   (d) Once you have verified that the balance is "pretty good", estimate uncertainty by adding or subtracting weight until the ruler has moved noticeably. Also move the distance in both directions until the ruler begins to swing.

   (e) Finally, verify your experimentally determined torque. Using your initial torque and the second weight's mass, calculate the second torque and the distance of the second weight.

2. For the second trial, you will calculate the theoretical magnitude and direction of the second torque *before* you test to see if your calculations are consistent with experiment.

   (a) Add a different weight to the ruler at a new distance. Write down the position of the fulcrum, the distance from the fulcrum to the weight, the mass of the weight, and the initial torque.

(b) Now calculate the magnitude and direction of the second torque. The direction tells you which side of the ruler to place the weight, why? Next pick a particular mass (hanger + weight) that is different from the first weight and calculate the distance from the fulcrum that you should place the mass to get the second, balancing torque.

(c) Verify your calculation experimentally.

7.3.2 Results

Again let's compare the theoretical and experimental results in a table, like the one below. Were there any disagreements? Why?

	Second Torque Magnitude (N·m)	Second Weight Distance (m)
Theory	0.54	0.054
Experiment	0.50 ± 0.05	0.057 ± 0.012

8 Fluids: Venturi Meter and Artificial Heart

8.1 Introduction

The aim of this lab is to give you some experience with fluid dynamics. Probably, the most important principle in physics that is used in medicine is Bernoulli's equation. Bernoulli's equation describes the pressures and velocities of fluids in a pipe, a model for blood in your arteries! Bernoulli's equation is:

$$p + \rho g y + \frac{1}{2}\rho v^2 = constant \tag{21}$$

where p is pressure, ρ is fluid density, g is the acceleration due to gravity, y is the height of the fluid, and v is the fluid velocity. If there is no elevation change, then the pressure in the pipe becomes directly related to the velocity in the pipe by:

$$p + \frac{1}{2}\rho v^2 = constant \tag{22}$$

This tells you that as pressure increases, velocity decreases and vice versa. So if you happen to have high blood pressure then that means that the velocity of the blood in your arteries will be smaller than normal, causing your heart to work harder and taking longer for oxygenated blood to reach your extremities.

Now let's think about atherosclerosis or hardening of the arteries where the artery becomes constricted. This is analogous to decreasing the radius, r, of a pipe. What happens to your blood pressure and velocity when this happens? First, let's look at velocity. Remember that the flow rate (Av, where A is the area of the pipe) through a pipe is constant. This is given by the equation of continuity:

$$A_1 v_1 = A_2 v_2 \tag{23}$$
$$r_1{}^2 v_1 = r_2{}^2 v_2 \tag{24}$$

From this equation we can see that as the radius decreases, the velocity in the pipe goes up. According to Bernoulli's equation a velocity increase would lead to a pressure decrease. At first this may seem counter-intuitive since atherosclerosis increases the likelihood that the artery will burst, causing a heart attack or stroke. But the artery does not burst due to the blood pressure in the constriction, it bursts because of the pressure the plaque exerts on the arterial wall or because a completely blocked artery would create an increase in blood pressure in the chamber before the build-up, akin to filling up a water balloon until it burst.

8.2 Experiment 1: Venturi Meter

Here we will use a venturi meter to measure velocity flow rate, Av. A venturi meter is a tube with a constricted throat that increases velocity and decreases pressure, as we described above. They are generally used to measure flow rates of fluids in a pipeline, like in a sewer system. They can also withstand very cold temperatures due to their lack of moving parts and are used in the oil pipelines in Alaska.

1. *Measure the radii of the chambers in the venturi meter*

   Measure the outer diameters (ODs) of the first two chambers at the top of the venturi meter with a caliper. Also measure the wall thickness and subtract this quantity correctly to get the inner diameter (ID). Then find the radii of the two chambers, r_1 and r_2. Estimate your uncertainty and be sure to correctly propagate it to the measurement of the radius.

2. *Setup the apparatus to measure the flow rate of an air line*

   Fill the bottom of the venturi meter a third of the way full with water. Then connect the bottom of the meter to the top of the meter. Keep the meter level with the table. Connect a 1/2" OD clear tube from the meter to the air line. Leave the other end of the venturi meter open to the atmosphere.

3. *Find the pressure difference inside the meter*

   Turn on the valve to the air line and watch the water in the tube. It should begin to rise or fall based on the pressure in the chamber. If the pressure in the constriction decreases relative to the first chamber, then it won't be able to "push" down as hard on the water relative to the first chamber. Write down in your notebook what the water levels in the bottom part of the meter look like. Make sure also to measure their heights (with the uncertainty). What does the water height say about the pressure in the chambers in the top part of the meter?

   Now calculate the pressure change between the constriction and the first chamber, $p_2 - p_1$. To do this, solve Bernoulli's equation for the pressure change on the water in the bottom of the meter. Find an expression for the pressure change in terms of the heights of the water. Record this pressure change and the calculated uncertainty.

4. *Calculate the flow rate*

   Using this pressure change you can calculate the velocity of the air inside the meter. To do this, solve Bernoulli's equation for the pressure change on the air in the top of the meter. You should find an expression that relates the pressure change to the velocity in the first chamber v_1, and to the radii of the first two chambers, r_1 and r_2. Since the pressure change on the air is the same as the pressure change on the water, you can now rearrange the equation to solve for the velocity. The flow rate is then just $A_1 v_1$. Record this quantity along with its uncertainty.

5. *Repeat the procedure for two more flow rates*

   Change the flow rate by turning the valve and remeasure the appropriate variables along with their uncertainties. You want to get to the point where you can do this rather quickly. I recommend typing all of the values into google (yes google is a calculator too) so that you only have to type in the height difference to get the velocity.

Q: Why can we only measure the flow rate out of the airline, not the actual velocity?
Q: Could you use the venturi meter to measure absolute pressure? Why or why not?

8.3 Experiment 2: Artificial Heart

In this section of the lab we will use an artificial heart to measure some physical properties of "blood" flow in the body. Our artificial body consists of a heart that pumps "blood" into the arteries; the "blood" then returns back to the heart via the veins. Here the artificial heart is a diaphragm pump, which is a rubber membrane with an attached handle and two one-way valves. This is a surprisingly good model of one side of the heart. The heart consists of two sides right and left. The right side takes the blood from the body through the veins and pumps it to the lungs, the left side takes the now oxygenated blood from the lungs and pumps it through the arteries into the body. Here we have simplified things by excluding the lungs so that only one pump is needed. Each side of an actual heart has two chambers and two one-way valves. The way the actual heart pumps is that the first chamber, the atrium expands filling with blood. As the atrium contracts the first one-way valve is opened and the blood enters the second chamber, the ventricle. The ventricle is expanding at this time, which also draws the blood into the chamber. Then the ventricle begins to contract closing the first one way valve and opening the second, pumping the blood out of the heart. Our artificial heart is simplified so that it doesn't include the atrium. After the heart, in our artificial body, we also have a system of arteries that can have different constrictions to model hardening of the arteries.

1. *Measure the flow rate of the artificial heart*

   To measure the flow rate of the artificial heart connect the venturi meter to the pump. Have one partner steadily pump the diaphragm while the other partner measures the water levels in the venturi meter. Use the water level measurement and the Bernoulli equation to calculate the flow rate. Is the flow rate constant? How does the flow rate of the artificial heart compare to the flow rate of an actual heart? (A normal heart pumps 5 L of blood per minute and 1 cm^3 = 1 mL.)

2. *Measure the blood pressure of the artificial heart*

   A blood pressure measurement is usually given as the systolic pressure over the diastolic pressure in mmHg. Systolic pressure occurs when the ventricle ejects the blood (in our case when the diaphragm membrane contracts) and represents the highest blood pressure that can occur. Diastolic pressure is the lowest pressure in the cycle and occurs when blood is entering the ventricle (in our case when the diaphragm membrane expands). Observe your artificial heart to see if this is true.

   Now let's measure artificial blood pressure. Disconnect the venturi meter so the part with three long tubes is not present. Then connect the PASCO pressure sensor to the first chamber. To connect the sensor to the computer see Appendix A. Pump the "blood" and look at the output of the pressure sensor. How does the pressure on the sensor relate to the "blood" pressure? Take the maximum pressure and convert it to mmHg as well as the minimum pressure. How does this compare to a normal blood pressure of "120 over 80" or 120 mmHg/ 80 mmHg?

3. *Measure the flow rates in arterial branches*

   In the body your arteries branch out carrying blood to different parts of the body. How does a branch affect the flow rate? Connect the venturi meter to one branch of the artificial artery. Keep all four branches open and monitor the flow rate. Now close one of the other branches and watch what happens to the flow rate. Next close two of the other branches, then all three of the other branches. What happens to the flow rate? Describe your observations using an equation.

4. *Measure how a constricted artery affects the blood velocity in an arterial branch*

   In the body some of your arteries may have atherosclerosis (hardening of the arteries), or some may just be bigger than others. To simulate different sized arteries you can turn the valves to constrict the flow so as to model a smaller artery. Play around with this. How does flow rate and velocity change in an arterial branch when one of the branches is constricted?

9 Waves and Oscillations: Simple Harmonic Motion and Standing Waves (Formal Report)

Make sure to read this lab in its entirety before coming to and beginning the lab. Before coming to lab, you must do the following exercises.
1. Substitute Eq. 27 into Eq. 26 (you will need to evaluate the second derivative of Eq. 27) and verify that Eq. 27 provides a solution to Eq. 26.
2. Find a relationship that expresses the period, T, as a function of the mass, m, and the spring constant, k.
3. Substitute Eq. 31 into Eq. 30 and show that Eq. 31 is a solution of Eq. 30 and from this determine the relationship between ω, g and L.

9.1 Introduction

9.1.1 Simple Harmonic Motion

In this experiment you will explore two situations which give rise to simple harmonic motion (SHM). The common characteristic of both situations is that an object is acted on by a force which increases **linearly** as the object moves away from an equilibrium position:

$$F_x = -kx, \tag{25}$$

where x is the displacement from the equilibrium ($F_x = 0$) position. Such a force is called a restoring force since the force tends to pull the object back to the equilibrium position. The farther the object is away from equilibrium, the larger the force pulling it back.

The motion of an object acted on by such a force is described by a differential equation of the form

$$\frac{d^2x}{dt^2} = -\omega^2 x, \tag{26}$$

where, as above, x is the displacement of the object from its equilibrium position, and ω is a parameter called the angular frequency. The solution of this differential equation, assuming that x is at its maximum value at $t = 0$, is

$$x(t) = A\cos(\omega t), \tag{27}$$

where the amplitude, A, is the maximum displacement of the system from its equilibrium position. An important property of simple harmonic motion is that ω is independent of A. The period of the oscillation is given by

$$T = \frac{2\pi}{\omega}. \tag{28}$$

In this laboratory you will be investigating the extent to which the oscillatory motion of a pendulum and of a mass on a spring can be described as simple harmonic motion.

Motion of a mass on a spring A spring provides one of the simplest examples of a linear restoring force. The simplest case occurs when the motion is horizontal and the mass slides on a horizontal frictionless plane. Having few such planes available, we will instead study the vertical motion of a mass suspended from a spring. The vertical position of the mass is an oscillatory function of the form

$$y(t) = y_0 + A\cos(\omega t) \tag{29}$$

This is similar to Eq. 27 except that the initial displacement, y_0, is not zero.

The Simple Pendulum For small oscillations, the angular displacement of the pendulum, θ, is described by the equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta. \tag{30}$$

Note that Eq. 30 has exactly the same form as Eq. 26. Hence, we may immediately conclude that the angular displacement described by $\theta(t)$ will execute simple harmonic motion

$$\theta(t) = \theta_0 \cos(\omega t) \tag{31}$$

where

$$\omega^2 = \frac{g}{L}. \tag{32}$$

9.1.2 Standing Waves

In this experiment, you will set up standing waves on a string by mechanically driving one end of it. As you change the driving frequency, you will observe the phenomenon of resonance. As the driving frequency approaches the resonant frequency the string will start to vibrate with a large amplitude. You will in effect set up standing waves along the string that you can measure the wavelength and frequency of.

The theory (covered in lectures) relates the wavelength and frequency of standing waves to the speed of propagation of waves in the medium (the string, in our example):

$$v = \lambda f \tag{33}$$

where v is the speed of the wave, λ is its wavelength, and f its frequency.

Theory also relates the speed of propagation of waves in a medium to the properties of the medium. Typically, the wave speed is the square root of an elastic property of the medium such as a tension or stress divided by an inertial property such as mass density. In the case of waves on a string, the speed is given by

$$v = \sqrt{\frac{F}{\mu}} \tag{34}$$

where F is the tension in the string, and μ is its mass per unit length. (We reserve the letter T for period, and hence don't use it for the tension in the string here). In the lab, you will choose

one value of F, and make a measurement of the wavelength of the standing waves for at least one resonant frequencies. Other groups will measure the same properties at different tensions. For a given tension, your data will allow you to investigate Eq. 33. The calculated average value of the velocity for each of the different tensions will permit a test of Eq. 34.
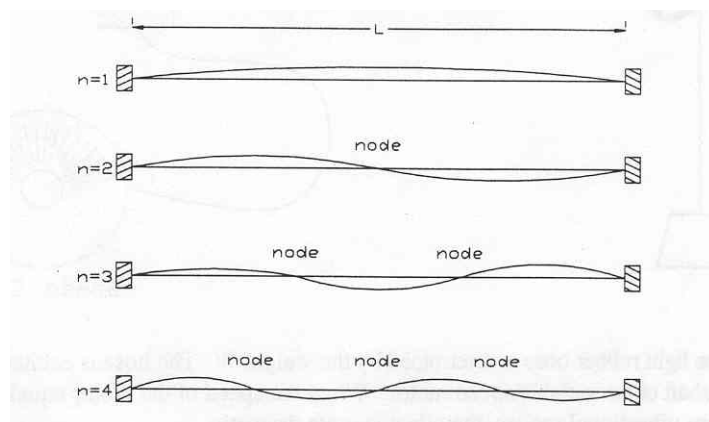
Figure 15: Standing waves on a string. n=1 corresponds to the fundamental mode; n>1 are harmonics.

When the string is held (more or less) fixed at the two ends, the standing wave patterns shown in Figure 15 are possible. The places where the wave disturbance is zero are called the nodes and the places where the disturbance is a maximum are called anti-nodes. One may think of a standing wave as the sum of two traveling waves, one traveling to the right and the other to the left. The distance between two successive nodes (or two successive anti-nodes) in a standing wave is half the wavelength of either of the traveling waves that comprise the standing wave. Clearly the fixed ends of the string are always nodes. With that constraint, the longest wavelength standing wave corresponds to having an *anti-node* $n = 1$ in the figure, and is variously called the fundamental or the "lowest note" or the lowest mode of the string, the latter terms coming from the fact that the frequency is lowest for these standing waves. The higher n values correspond to harmonics of the fundamental, the overtones. Mathematically, it is not hard to check that

$$\lambda_n = \frac{2L}{n}. \tag{35}$$

The frequencies are given by

$$f_n = \frac{v}{\lambda_n} = nf_1. \tag{36}$$

We should note that the string does not have to vibrate so that it has only one of these basic patterns of standing waves. In general, the excitation of a string (like that of a stringed instrument) will produce a mixture of these different patterns with different amplitudes simultaneously. The mix of overtones, among other features, distinguishes a violin from a piano or other instruments all playing the same "note", and lends music a richness that is hard to capture by our simple analysis.

Still, understanding of the pure modes allows us to analyze even highly complicated mixtures of modes.

9.2 Experiment 1: Mass on a Spring

1. *Study and measure the period*

   Choose one of the two conic shaped masses provided and attach it to the spring. Measure the mass. Note: in is important to record the mass of the spring itself. In all calculations involving the oscillating mass, use the *total* mass:

   $$m_{tot} = m_{hanging} + \frac{1}{3} m_{spring} \tag{37}$$

   Measure the period of oscillation.

   Q: Why is it better to determine the time for 50 oscillations than for just 5?

   Carry out measurements for at least different 3 amplitudes to see if the period is independent of the amplitude of the motion as predicted. Make sure you stay in the limit of small oscillations (< 15 cm). Record the values of the amplitudes you use.

2. *Determine the spring constant*

   From your period measurements, determine the spring parameter, k.

3. *Repeat for the second mass*

   Attach the second mass to the spring and measure the period of its motion. Determine the spring parameter, k, from these measurements.

   Q: For the two masses you used above, what *should* the ratio of the periods be? Calculate the ratio of the measured periods, and compare it with the predicted ratio. Do they agree within uncertainty? If not, why not?

4. *Hooke's Law*

   To test Hooke's Law make a series of measurements of the stretch of the spring, x, vs. the applied force, $F = mg$. Hang a mass hanger on the end of the spring. Starting with 100g, add mass to the hanger and measure the length of the spring. Make several measurements between 100-800g. Plot F vs. x. Describe qualitatively the relation between the weight and length of the spring.

   Q: What range of values does the spring obey Hooke's Law?

   Determine the spring constant k using regression analysis. Compare the value obtained with the values determined in steps 2 and 3.

5. *Measure the parameters using a motion sensor*

Set up a motion sensor to observe the position of an oscillating mass as a function of time. In the equilibrium position, the mass (about 450 g) should be about 70 cm above the sensor. You may have to try more than one combination of sample interval and number of samples before finding a good one.

(a) Use the motion sensor to measure the position of the mass for a ∼5 second interval after you have displaced the mass about 5 cm from its equilibrium position and carefully released it. Do not start the motion sensor until after you have released the mass and moved well away from the experimental apparatus. Make certain no one is leaning on the table during the measurement.

(b) Look at the graph of position vs. time. If you see 3 or so cycles of a relatively noise-free sine curve, you are ready to export the data to a spreadsheet. See Appendix A for instructions on exporting data. Make sure you export position, velocity and acceleration vs. time data. If the data are noisy, repeat the measurement.

(c) Construct a graph which has plots of a, $10x$, and $10v$ versus t. Enlarge the graph, label axes, etc. and put in grid lines. [Note: The factors of ten are used to ensure that all three plots are visible.] You may also find it useful to use lines for the x vs. t and v vs. t graphs, whereas a symbol plot is probably better for the a vs. t plot.

(d) Include the graph with your **Formal Report**. Discuss the relative phases of the x vs. t, v vs. t, and a vs. t graphs.

(e) By Newton's second law, F should be directly proportional to a. This gives us convenient way to measure k, since $ma = -kx$, and the motion sensor gives us both x and a. Make a plot of a vs. x, which you should include in your formal report. From the slope of the best-fit line, find k. Does this value agree with your previous values?

9.3 Experiment 2: Simple Pendulum

1. *Study the period of a simple pendulum*

Choose one of the masses and make a simple pendulum of about 1 meter length. Determine the period of oscillation for small oscillation amplitudes ($< 20°$). Carry out at least three measurements to see if the period is independent of the amplitude. Record the values of the amplitudes you use. Now make a pendulum of the same length with the *second* mass and once again measure the period.
Q: How do your results compare with the theoretical dependence of period upon mass?

2. *Calculate g using a simple pendulum*

Determine g by measuring the period of the *longest* pendulum you can conveniently make. To do this, we will have set up a pendulum in the central stairwell from the fifth-floor landing to the bottom of the stairwell, 4+ floors below. Give some thought to exactly what length you need to measure, what are the end points?

9.4 Experiment 3: Standing Waves

For this section of the lab you will take data on the period and wavelength of standing waves on a string with a particular tension. Each group will take data at a different string tension and then at the end we will compile all of the results to extract the velocity along the string.

1. *Change the tension, F, in the string*

   Choose a hanging mass between 200 g and 800 g. Make sure that the masses are placed securely on the hanger. Note the total mass along with uncertainty. Calculate the tension along with its uncertainty.

2. *Adjust to $n = 1$ and determine f_1 and T*

   Carefully adjust the drive frequency so that the string is vibrating in the fundamental mode. Note the frequency on the function generator. Repeat the frequency measurement at least twice, each time moving away from resonance and moving back. Share the task of judging when you have hit resonance equally with your lab partner(s). Use the range of frequencies as a measure of experimental uncertainty. Now calculate the period, T, and its uncertainty.

3. *Measure $\lambda \pm \Delta\lambda$*

   Measure the distance between successive nodes. This will allow you to calculate the wavelength. Estimate how well you can locate the nodes, and hence the experimental uncertainty in the wavelength.[2]

4. *Analysis*

   You will find the velocity of the waves on the string graphically. If we rearrange Eq. 33, we can find an expression that treats λ and f_1 as two independent variables:

$$\lambda = \frac{v}{f}$$
$$\lambda = vT \tag{38}$$

   The new expression now predicts that a plot of λ vs. the period T will be a straight line with slope equal to v.

   (a) Enter your data into an Excel spreadsheet. Make sure to properly label each column including *units*. Also include error bars.

   (b) Generate a plot of λ vs. T.

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[2]For $n = 1$, the location of only one node is well defined. The other end of the string is being driven and thus has no node. Notice, however, that as you move along the string in the direction of the driver, the amplitude of the vibration is decreasing. You can still estimate the wavelength for the fundamental mode by extrapolating to the point where the the node *would* exist *if* the string continued without interruption. This point will often be very near the rod which the string is tied to, but its exact position is often difficult to pinpoint.

(c) Use the regression analysis tool to find the best value and *uncertainty* of the slope.

(d) Now find the velocity given the tension, F, and the mass per unit length of the string, μ (this quantity will be given to you by the instructor).

(e) Compare your results from the regression analysis to the value of the velocity you just calculated.

9.5   Formal Lab Report

A formal lab report is to be written based upon your findings. Follow the guidelines for formal reports in Appendix D. The report should include the following:

1. Discuss your results for the mass on the spring experiment.

2. Discuss your results for the pendulum experiment.

3. Discuss the class results for the standing waves experiment.

4. For both cases *you* must decide what is important in your results and how to present that information (tables, graphs, etc.). Be sure to include some discussion of how experimental uncertainties affect your results.

A   Using the PASCO Motion Sensor and DataStudio

Several labs make use of the motion sensor and computer. This appendix outlines the general instructions for use of this system.

A.1   Setting up the Motion Sensor and DataStudio

If the computer is not turned on by the beginning of the lab, please do so. Log on to the computer using your campus username and password. Connect the motion sensor to the PASPort USB Link, and then connect the USB Link to a USB port of the computer. A window should pop up with the title `PASPortal`. Select "Launch DataStudio."

An alternative way is to select `Start > All Programs > DataStudio > DataStudio` from the taskbar. Click the `Setup` button. The Experiment Setup window will open. Select `Connect`. A message window will open. Select `Choose`. From the list of interface choices, choose `PASPORT` and then click `OK`.

A.2   Configuring the Motion Sensor

To configure the sampling rate of the motion sensor, start by clicking the `Setup` button. The Experiment Setup window will open. You can select what kind of data will be displayed by clicking the boxes next to `Position`, `Velocity`, and `Acceleration`. Choose the sample rate by clicking the "+" or "-" buttons.

To choose between manual or automatic sampling, select the `Options...` button. A "Sampling Options" window will open. Click the `Automatic Stop` tab. If you wish to manually stop the data sampling, make sure the radio button `None` is selected. If you wish to sample data for a fixed amount of time, select `Time` and then enter the duration of the sample in seconds.

A.3   Taking a Data Sample

To take a data sample, simply click `Start`. If you are using the manual sampling mode, the data sampling will continue until you click `Stop`; otherwise, the sampling will stop at the end of the set time interval. The data are recorded in "Runs." You can take another sample by clicking `Start` again, a new sample will be recorded. The runs are numbered sequentially.

A.4   Exporting Data

To export data select `File > Export Data...` from the menu. A window will open with a list of runs. Select the run you wish to export and click `OK`. A file manager window will open by default in your "My Documents" folder. Type the name of the file in the "filename" box and then click `Save`. The data will be saved with a ".txt" extension.

A.5   Opening Data in Excel

Start Excel. From the menu select `File > Open...` From the file manager window select the file that you exported from DataStudio. If you do not see the file listed, go to "Files of types" and select either `All Files` or `Text Files`. Select the data file and click `Open`. The Text Import Wizard will open. Select the `Delimited` radio button, then click `Finish`.

B Keeping a lab notebook

Keeping a good lab notebook seems like a simple and obvious task, but it requires more care and thought than most people realize. It is a skill that requires consistent effort and discipline and is worth the effort to develop. Your lab notebook is your written record of everything you did in the lab. Hence it includes not only your tables of data, but notes on your procedure, and your data analysis as well. With practice, you will become adept at sharing your time fairly between conducting the experiment and recording relevant information in your notebook as you go along.

You want all this information in one place for three main reasons, and these reasons continue to be valid even after you leave the introductory physics laboratory. First, your lab notebook contains the information you will need to write a convincing report on your work, whether that report is for a grade in a course or a journal article. Second, you may need to return to your work months or even years after you have finished an experiment. It is surprising how often some early experiment or calculation is important in your later work. Hence, you need a reasonably complete account of what you have done. Third, your notebook is the source to which you turn in case someone questions the validity of your results. You should write as much detail as needed for someone, with only the lab manual and your lab notebook, to reproduce the experiments you performed and the calculations you did exactly.

Your notebook therefore serves two purposes that may not be completely compatible with each other. On the one hand, you should write things down pretty much as they occur and before you have a chance to forget them, so that you have a complete record of your work in the lab. On the other hand, your notebook should be reasonably neat and well-organized, partially so you can find things and partially so that if anyone questions your results, not only will they be able to find things, but the layout of your notebook will suggest that you investigated the problem carefully and systematically.

You should use a bound lab notebook (that is, not a loose leaf notebook). So-called quadrille notebooks (with rectangular grids on each page) are particularly handy for making graphs and tables. If you wish to add a graph done on a computer to the notebook or a graph done on regular graph paper, you may simply tape or glue the graph into your notebook.

Next we will discuss some of the information that goes into your lab notebook.

B.1 Title, Date, Equipment

You should begin each new experiment on a fresh page in your notebook. Start with the date and a brief title for the experiment—just enough to remind you what that section of your notebook is about. Identify large pieces of equipment with the manufacturer's name, the model, and the serial number. With this information, you can repeat the experiment with the identical equipment if for some reason you are interrupted and have to return to the equipment much later. Or, if you are suspicious of some piece of equipment, having this information will let you avoid that particular item.

B.2 Sketch of the Setup

Make a quick sketch of the setup, including labels that define all of the important parts. Also, if there are variables that are related to the sketch please include them in the drawing. For example, if there is a distance between a mass and a pivot point in the setup that is called r, denote r in your diagram with arrows between the relevant points.

B.3 Procedure

You won't be expected to copy the procedure printed in your lab manual into your notebook, but be sure to indicate where the procedure can be found. It is essential that you note any deviation from the procedure described in the manual. Write in complete sentences and complete paragraphs. This is part of the discipline required for keeping a good lab notebook. Single words or phrases rapidly become mysterious, and only with a sentence or two about what you're measuring, such as the period of the pendulum as a function of length, will you be able to understand later on what you did. Give more details where necessary, if for example the lab manual does not give a more detailed procedure or if you depart from the procedure in the manual.

B.4 Numerical Data

When recording numerical data, keep your results in an orderly table. You should label the columns, and indicate the units in which quantities are measured. You should also indicate the uncertainty to be associated with each measurement. If the uncertainty is the same for all data of a certain set, you can simply indicate that uncertainty at the top of the column for those data points.

You will need at least two columns, one for the independent variable and one for each dependent variable. It's also good to have an additional column, usually at the right-hand edge of the page, labeled "Remarks." That way, if you make a measurement and decide that you didn't quite carry out your procedure correctly, you can make a note to that effect in the "Remarks" column. (For example, suppose that you realize in looking at your pendulum data that one of your measurements must have timed only nine swings instead of ten. If you indicate that with, say "9 swings?" you could justify to a suspicious reader your decision to omit that point from your analysis.)

B.5 Sequences of Measurements

You will often be performing experiments in which you have two independent variables. Usually in such experiments you fix the value of one independent variable and make a series of measurements working through several values of the other variable. Then you change the value of the first variable and run through the measurements with the other variable again; then you change the first independent variable again, make another set of measurements, and so on. It's usually easier to set up this sort of sequence in your notebook as a series of two-column tables (or three columns with "Remarks") rather than a big rectangular grid. "Title" each table with the value of the independent variable that you're holding fixed, and keep the format of all the tables the same.

B.6   Comment on Results

Once you have completed the experiment and performed any necessary calculations in the notebook, you should look back to the main goal and write down to what extent it was achieved. If, for example, you were making a measurement of *g*, you should include a clear statement of the value of *g* along with its uncertainty. Be aware that there are often secondary goals as well (to become familiar with a particular physical system or measurement technique, for example). Comment on your success in attaining these goals as appropriate. This serves as a statement of conclusion and gives you the chance to make sure the lab was completed thoroughly and to your satisfaction.

B.7   Guidelines to Keeping a Good Notebook

1. Bound Notebook: No spiral bound, loose leaf or perforated page notebook. Lab notebooks are a *permanent record* of the work done in lab. The integrity of the notebook should not be comprised by tearing out pages.

2. Keep a record. Write down names, title, time, places, and dates.

3. Be generous with the use of pages. Start each experiment on a fresh page. Leave some blank pages between experiments in case you need to add tables or graphs.

4. **Always** write in ink and **never** write in pencil. You should also never erase calculations, data, comments, etc., because the original data, calculation and so on, may turn out to be correct after all, and in any case, you want to keep a complete record of your work, even the false starts. If you believe that a calculation is wrong then draw one single line through the calculation and make a note in the margin.Do not erase or scribble out possible mistakes.

5. Define terminology and variables with units.

6. Sketch the experimental setup. Label the relevant parts and indicated measured quantities. List equipment used.

7. Be complete. Every experiment has a introduction (pre-lab note), procedure, analysis, results and summary. Make sure all tables, graphs, diagrams and calculations are in your notebook for reference.

8. Annotate! Each section of the experiment should begin with a paragraph discussing what the section is about. Do not leave it to a reader to guess.

9. Be organized, neat and legible. The reader should not to have struggle to decipher your notes. Don't be cryptic.

10. Answer all questions and exercises. Use your answers to the questions to build the discussion of your formal report or exit interview. Always answer questions and exercise from a physics point of view.

C   Graphical Presentation of Data

C.1   Introduction

"Draw a picture!" is an important general principle in explaining things. It's important because most people think visually, processing visual information much more quickly than information in other forms. Graphing your data shows relationships much more clearly and quickly, both to you and your reader, than presenting the same information in a table.

Typically you use two levels of graphing in the lab. A graph that appears in your final report is a "higher-level" graph. Such a graph is done neatly (and almost always with a graphing program), following all the presentation guidelines listed below. It's made primarily for the benefit of the person reading your report. "Lower-level" graphs are rough graphs that you make for your own benefit in the lab room; they're the ones the lab assistants will hound you to construct. These lower-level graphs tell you when you need to take more data or check a data point. They're most useful when you make them in time to act on them, which means that you should get in the habit of graphing your data in the lab while you still have access to the equipment. (That's one reason, in fact, that we recommend that you leave every other sheet in your lab notebook free, so you can use that blank sheet to graph your data.) In graphing your data in the lab, you don't need to be too fussy about taking up the whole page or making the divisions nice. You should label the axes and title the graph, though.

Flaky data points show up almost immediately in a graph, which is one reason to graph your data in the lab. Skipping this low-level graphing step can allow problems in the data collection to propagate undetected and require you to perform the experiment again from the beginning. Graphing each point as you take it is probably not the best idea, though. Doing so can be inefficient and may prejudice you about the value of the next data point. So your best bet is to take five or six data points and graph them all at once.

Graphing your data right away also flags regions in your data range where you should take more data. Typically people take approximately evenly-spaced data points over the entire range of the independent variable, which is certainly a good way to start. A graph of that "survey" data will tell you if there are regions where you should look more closely; regions where you graph is changing rapidly, going through a minimum or maximum, or changing curvature, for example. The graph helps you identify interesting sections where you should get more data, and saves you from taking lots of data in regions where nothing much is happening.

C.2   Analyzing your Graph

In this course, "Graphical data analysis" is usually a euphemism for "find the slope and intercept of a line", though in the real world that will not be the case. You will find this semester that you spend a lot of time redrawing curves by employing the "method of straight line graphing" so that they turn into straight lines, for which you can calculate a slope and an intercept. This process is so important that, although we have a fond hope that you learned how to do this in high school, we're going to review it anyway.

Presumably you have in front of you some graphed data that look pretty linear. Start by drawing in by eye the line that you think best represents the trend in your data. An analytical procedure exists to draw such a line, but in fact your eyeballed line will be pretty close to this analytically-determined "best" line. Your job now is to find the slope and intercept of that "best" line you've drawn.

Next we tackle the question of finding the slope and intercept of that line. As usual, we will assume that the line is described by the equation

$$y = mx + b \tag{39}$$

where m is the slope of the line and b is the y-intercept.

Two points determine a line, and a line is also described completely by its slope and intercept. (This should make a certain amount of sense. You put in two pieces of information, you get out two pieces of information.) Your first task is therefore to choose two points on your line. These two points describe the line, so they need not (and most likely *will* not) be data points. They should be far apart on the graph, to minimize the effects of the inevitable experimental uncertainty in reading their locations from the graph paper. The two points should also be located at easy-to-read crossings on the graph paper. Mark each of those points with a heavy (but not too large) dot and draw a circle around the dot. Read the coordinates of each point off the graph.

The slope of the line is defined as the change in y (the vertical coordinate) divided by the corresponding change in x (the horizontal coordinate). (You may know this in some other form, such as "rise over run.") To calculate the slope, use

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}, \tag{40}$$

substituting your values for the points (x_1, y_1) and (x_2, y_2). For example, if your points are $(1.0 \text{ s}, 8.8 \text{ m/s})$ and $(6.0 \text{ s}, 46.3 \text{ m/s})$, then $m = 7.5 \text{ m/s}^2$. (Notice that the units of m are the units of "rise-over-run.") Now that you have the slope, find the intercept from

$$\text{intercept} = b = y_1 - mx_1. \tag{41}$$

That is, you can read b directly off off the graph, or you can use the slope and one point to determine b. Use either point for (x_1, y_1). Both lie on the line, so either will work. In the example above, we get that $b = 1.3 \text{ m/s}$. (Notice that the units of b are those of the y variable.)

Once you have determined the values of m and b from the graph, you can quote the equation for your straight line. For example, if $m = 7.5 \text{ m/s}^2$ and $b = 1.3 \text{ m/s}$, then the equation of your straight line is

$$y = (7.5 \text{ m/s}^2)\text{x} + 1.3 \text{ m/s}. \tag{42}$$

This equation gives a **complete** description of the line and the job is done.

C.3 Uncertainty Bars

Individual data points plotted on any graph should include uncertainty bars (sometimes misleadingly called "error bars") showing the uncertainty range associated with each data point. You should show both vertical and horizontal uncertainty bars, if the uncertainties are large enough to be visible on the graph. (If they aren't large enough, you should mention this in your report so we don't think you've forgotten them.) You can draw uncertainty bars by indicating the "best guess" value (typically the measured value or average of several measurements) with a dot, and drawing an "I-bar" through the dot with its length indicating the range in the uncertainty. When you use Excel or the Graphical Analysis data analysis package, this step can be done for you — with severe limitations. Such a package will typically only determine error bars by considering the scatter of the individual data points about the best-fit straight line. While this is helpful in providing a consistency check for the data, it does not tell the complete story of the uncertainties in your data. That is, unless you use a more advanced feature of such an analysis program, it has no way of knowing about the uncertainties that were inherent in your measured values because of the measurement apparatus. Only you can decide how accurately you used the meter stick, or how quickly you were able to react when starting and stopping a stopwatch. You will not always be expected to put error bars on all of your plotted points, but you should know how it is done and be able to apply it to the first lab.

C.4 Graphical Presentation Guidelines

Use these guidelines for "higher-level" graphs.

1. Scale your axes to take the best possible advantage of the graph paper. That is, draw as large a graph as possible, but the divisions of the graph paper should correspond to some nice interval like 1, 2, or 5 (times some power of 10). If you have to make the graph smaller to get a nice interval, make it smaller, but check that you've picked the nice interval that gives you the largest graph. Making the graph large will display your data in as much detail as possible. When using log-log or semi-log paper, choose paper with the number of cycles that gives the largest possible graph.

2. The lower left-hand corner need not be the point $(0,0)$. Choose the range of values for each axis to be just wide enough to display all the data you want. If $(0,0)$ does not appear on the graph, it's customary (but not necessary) to mark the break in the axis with two wavy lines ($\approx$).

3. Mark the scale of each axis (the number of units corresponding to each division) for the entire length of the axis.

4. Label both axes, identifying the quantity being plotted on each axis *and* the units being used.

5. Give each graph a title or provide a figure caption. The title should summarize the information contained in the axes and also give any additional information needed to distinguish this graph from other graphs in the report.

6. Give each graph a number (e.g., "Figure 2"), which you can use in the body of the report to refer quickly to the graph.

7. If you calculate the slope and intercept of the graph from two points (rather than using linear regression), indicate the two points you used on the graph. Draw the line through the two points, label it "Best-fit line" (or something similar), and give its slope and intercept on the graph in some large clear space.
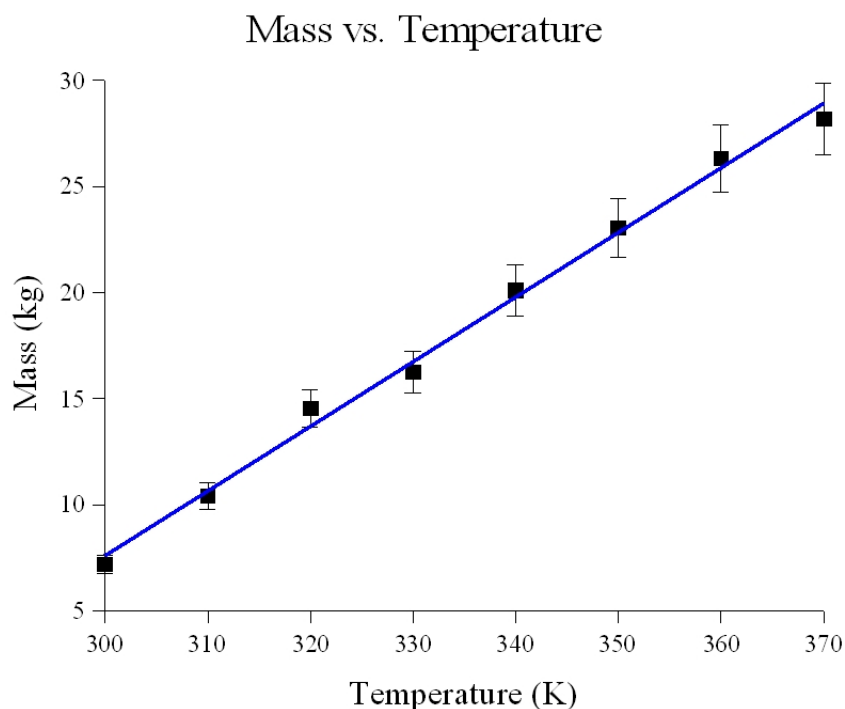
Figure 16: Mass vs. Temperature
This is a sample graph illustrating all the features of a "high-level" graph. The solid line represents the best fit to the sample data.

C.4.1   Graphing Checklist

- Axes scaled correctly with divisions equal to "nice" intervals (1, 2, 5 or 10);

- Graph drawn to as large a scale as possible;

- Scales on axes labeled for entire length;

- Axes labeled, including units;

- Graph titled and numbered; and

- Points used to calculate slope and intercept clearly marked, if that method is used.

D   Guidelines for Formal Laboratory Reports

The formal lab report should be a complete presentation of your work on the experiment. It should be written for someone who has a physics background equivalent to this course, *but who does not know anything about the experiment and the measurements you carried out.* There are three principal components to every formal report:

- *Format*: the organization and presentation of the report.

- *Composition*: the style in which the report is written.

- *Content*: the subject matter of the report.

All three are essential for writing a complete and self-consistent report. The purpose of the reports is to test both your analytical skills and your writing skills at communicating physical concepts. It must be emphasized: formal reports are short papers, not questionnaires or "fill in the blank". The reports will be graded with same importance as papers in other course. All components of the report will critiqued in the grading.

D.1   Format

The report should be formatted in a way that clearly presents all the relevant information to the reader: text, equations, figures, etc. Some standard report formatting include:

- Typeset using a word-processor.

- Lines double spaced.

- 12 point Times New Roman font.

- 1" left and right margins.

- Text is full justification.

- Reports are no more than 6 pages long.

The organization of the report is crucial. The reader anticipates a particular order for the report to be presented. Deviation from that order will mislead or confuse the reader. The report should be organized as follows:

1. **Title:** The title should be a simple descriptive phrase, centered at the top of the first page of the report. Also include your name, the date, the lab section and the name(s) of your partner(s).

2. **Introduction:** The introduction is a short, single paragraph statement of the experiment. What is the purpose, the *main* goal, of experiment and why is the experiment a worthwhile means of exploring a particular physical concept?

3. **Theoretical background:** The theoretical background should state what the underlying physics of the experiment is. What the theory predicts, what assumptions have been made, and how the experiment relates to the theory of the physics being studied. Terminology specific to the experiment should be defined. Often, the theory can be best expressed analytically in the form of an equation. Define the quantities to be determined and how they are related to the directly measured quantities.

4. **Experimental technique:** The experiment technique should be a detailed *narrative* of the experimental procedure. *What* was measured and *how* was it measured? Include a simple diagram of the apparatus whenever possible. Indicate the primary sources of measurement uncertainty. Give numerical estimates of uncertainties associated with each directly measured quantity.

5. **Data, analysis, and results:** Display the data in one or more appropriate forms (tables, graphs, etc). Discuss how the final results are obtained. Give estimates of the uncertainty of the results based upon measurements uncertainties. Be sure to include some discussion of experimental uncertainties and how those uncertainties affect the evaluation of your results.

6. **Discussion of results and Conclusion:** The conclusion should reflect your overall understanding of the experiment, i.e., what have you learned about the particular subject of physics studied in the experiment? It should consist of a logical sequence of statements substantiated by the evidence presented in the report. Was the goal of the experiment accomplished? Were the experimental results *consistent* with theoretical expectations? That is, do they agree within the range of uncertainty? What are the implications of your results? It is good practice to restate any numerical results in the conclusion for easy review by the reader.

Other forms of information require specific formatting:

- **Equations:** Equations should be centered on separate lines from the text of the report. Each equation should be numbered, preferably along the right margin, for easy reference. An equation is often followed by a sentence that defines the variables in the equation. Equations are especially useful when stating the theoretical background of the report. Do not include long derivations of equations in the report. Instead, simply reference which equations are used in the derivation and give final result. Keep derivations in your notebooks.

- **Figures - Diagrams:** diagrams of experimental setup should be simple yet illustrative of the experimental setup and apparatus. The relevant parts should be labeled and the relevant measured quantities indicated. Each diagram should have a figure number (Figure 1:) and caption below the diagram. The caption should be a concise description of the figure and any important parts. Use drafting tools like rules, protractors or applications like MS Paint to draw diagrams. Do not draw free hand diagrams.

- **Figures - Graphs:** The title of a graph should clearly indicate which two quantities are plotted. The title convention for a graph is Y (vertical axis) vs. X (horizontal axis). The

axes should be labeled and include units. Graphs use the same convention of numbering and captions as diagrams (Figure 1:). The caption should be a brief description of the graph and the quantities plotted. Adjust scale of axes so data points fill the whole graph. Empty space is a waste. For more details refer to Appendix **??**, Graphical Presentation of Data.

- **Tables:** Data tables should be organized in columns. The head of each column should be labeled and include units. If the quantities in the column all have the same uncertainty, then the uncertainty can be indicated at the head of the column as well; for example, Time (± 0.001 sec). Each table should have a descriptive title, starting with a number (Table 1:) for easy reference. Do not split tables across pages of the report. Do not include long tables like the data tables from motion sensor. Keep those tables in a spreadsheet.

D.2 Composition

A formal lab report is paper, similar to other papers written in other courses, and should follow the accepted conventions of composition. The report should be written in narrative style. Correct spelling, grammar, punctuation and syntax are essential.

Always make use of resources when writing: dictionary, thesausus, handbook. Probably the best handbook for college writing is Strunk and White *The Elements of Style*, 4th ed. It is particularly good in its no nonsense approach to writing, and it has been a standard handbook for collegiate writing for almost 100 years, and it is the standard handbook for this course. Amherst College Writing Center also has links on its webpage to several online resources.

The style of scientific writing is definitive, concrete and fact-based. It is not poetic, literary, sarcastic or opinionated. Specific styles of writing the *should not* be used in your report:

- Editorial - expressions of opinion or commentary. The language should be simple and substantive; based on the evidence presented in the report. It should not be an expression of how the writer feels about the experiment.

- Bombastic - inflated or grandiose language. Reports are not exercises in creative writing. The purpose of the report is to educate the reader *not* impress the reader. Keep the language simple.

- Verbiage - excessively wordy but conveying little or no information. Do not ramble.

- Abstract - void of concrete, real world meaning. Use physical terminology and use it correctly.

- Circumlocution - excessive use of words to explain a concept or idea. Do not over explain the experiment from first principles. Be specific to the subject matter.

Standard conventions include:

- **Write in narrative prose, not outline form.** This is especially true when writing sections of the report like the experimental technique. Do not recite the procedure outlined in the lab manual.

- **Use simple, complete sentences.** Sentences are expressions of one complete thought, fact or idea. The simpler the sentence, the better. Avoid excessive use of qualifiers, modifier and subordinate clauses and phrases. Get to the point and stay on topic.

- **Write in paragraph style.** The paragraph is the building block of the report. Each paragraph should address one topic of the report. Indent!

- **Use correct terminology, spelling and grammar.** There are some words in everyday language that have specific meaning in the context of physics. Make sure the terminology is consistent with the subject matter of the report. Always use the standard spelling of words.

- **Write formally; do not use slang or colloquialisms.** Do not write in a casual manner; for example, the word "plug", as in, "I plugged the numbers into the equation.." This is a sloppy, lazy style of writing. Formal reports should be written in a formal style.

D.3 Content

The content of the report addresses the subject matter; the principles, ideas and concepts the report is about. Since we believe physics is a logical, self-consistent science, the content of the report should be logical and self-consistent as well.

- **Clarity**: the underlying principles are clearly articulated, all relevant terminology is defined. You should be specific in the language used. Avoid vague or ambiguous statements.

- **Completeness**: all elements of the report are present. Missing or omitted content will mislead or confuse the reader.

- **Conciseness**: specific and to the point. The writer should avoid redundant, irrelevant or circuitous statements. Stay on topic.

- **Consistency**: all elements of the report direct the reader to a single, logical conclusion. Avoid illogical, erroneous, unsubstantiated, specious, irrelevant statements and contradictions.

- **Continuity**: all elements of the report follow a logical order. The discussion is constructed in a sequential manner. Avoid incoherent, disorganized statements.

D.4 Questions and Exercises

In some experiments specific questions and exercises will be asked. The purpose of the questions and exercises is to motivate the discussion. Questions and exercises should be answered within the body of the report and always from a *physics* point of view. The report is incomplete without answers to questions and exercises.

D.5   Some general writing guidelines

- Do not assume too much about the reader's knowledge of the experiment. It's your responsibility to explain the subject matter to the reader. Assuming the reader already understands the subject and not providing a complete explanation makes the report seem disjointed.

- Proofread! Spelling and grammatical errors are easy to fix; otherwise the report appears sloppy. Make use of resources: dictionary, thesaurus, *Strunk and White*, etc.

- Check the Units, Significant Figures and Uncertainties! Values without units are meaningless. Make sure all the values have the correct units. Checking the unit of a value is also a good cross check of a calculation. For example, if a calculation is solving for a velocity (m/sec) but the solution has units in kilograms, then there may be an error in the calculation.

- Do not mix together discussion of theory, procedure and analysis. Use subheadings; for example, Theory, Analysis, Conclusion, to keep the report organized and the reader's attention focused.

- Do not over explain the experimental setup. A well drawn diagram of the setup is better than a lot of prose. Keep the topic of the report on the *physics*.

- Do not over explain the use of instruments like computers, calculators or software like Excel©. Computers and softwares are only tools. Keep to the *physics*.

- Always present a result in the form $x_{meas} = x_{best} \pm \Delta x$. It is nonsense to present the best value separate from the uncertainty.

- Do not wrap text around figures, tables or equations.

- Do not include long tables of data in report; especially data from motion sensors. Long tables are boring to the reader and the data are better presented in a graph.

- Do not include summary Output Page from Regression Analysis in report. Keep Output Page in your notebook. Only extract the necessary values for you report.

- Do not include long calculations and algebra derivations. Reference the equation that is evaluated and state result. Only give derivations if it is asked for in a specific exercise. Keep a record of any detail calculations in your lab notebook.

- Use drafting tools like rules, protractors, compasses or or applications like MS Paint to draw diagrams. No free-hand diagrams.

- DO NOT DOWNLOAD FIGURES FROM INTERNET! Produce your own figures.

- Ask for help. Don't be afraid to ask questions if you are unclear on something. There is no need to guess what we want in the report.

E Experimental Uncertainty Analysis

An intrinsic feature of every measurement is the uncertainty associated with the result of that measurement. No measurement is ever exact. Being able to determine easily and assess intelligently measurement uncertainties is an important skill in any type of scientific work. The measurement (or experimental) uncertainty should be considered an **essential** part of every measurement.

Why make such a fuss over measurement uncertainties? Indeed, in many cases the uncertainties are so small that, for some purposes, we needn't worry about them. On the other hand, there are many situations in which small changes might be very significant. A clear statement of measurement uncertainties helps us assess deviations from expected results. For example, suppose two scientists report measurements of the speed of light (in vacuum). Scientist Curie reports 2.99×10^8 m/sec. Scientist Wu reports 2.98×10^8 m/sec. There are several possible conclusions we could draw from these reported results:

1. These scientists have discovered that the speed of light is not a universal constant.

2. Curie's result is better because it agrees with the "accepted" value for the speed of light.

3. Wu's result is worse because it disagrees with the accepted value for the speed of light.

4. Wu made a mistake in measuring the speed of light.

However, without knowing the uncertainties, which should accompany the results of the measurement, we cannot assess the results at all!

E.1 Expressing Experimental Uncertainties

Suppose that we have measured the distance between two points on a piece of paper. There are two common ways of expressing the uncertainty associated with that measurement: absolute uncertainty and relative uncertainty. In both ways the measured quantity is expressed in the form:

$$x_{measured} = x_{best} \pm \Delta x \tag{43}$$

Here x_{best} is the best measured value, usually from an average of a set of measurements, and Δx is the uncertainty in the best measured value. The measurement is always a *range* of values, not just the best value.

E.1.1 Absolute Uncertainty

We might express the result of the measurement as

$$5.1 \text{ cm} \pm 0.1 \text{ cm} \tag{44}$$

By this we mean that the result (usually an *average* result) of the set of measurements is 5.1 cm, but given the conditions under which the measurements were made, the fuzziness of the points, and the refinement of our distance measuring equipment, it is our best judgment that the "actual" distance might lie between 5.0 cm and 5.2 cm.

E.1.2 Relative (or Percent) Uncertainty

The relative uncertainty is defined:

$$f_x = \frac{\Delta x}{|x_{best}|} \tag{45}$$

We might express the same measurement result as

$$x_{measured} = x_{best} \pm f_x \tag{46}$$

For example:

$$5.1 \text{ cm} \pm 2\%. \tag{47}$$

Here the uncertainty is expressed as a percent of the measured value. Both means of expressing uncertainties are in common use and, of course, express the same uncertainty.

E.1.3 Graphical Presentation: Uncertainty Bars

If we are presenting our data on a graph, it is traditional to add uncertainty bars (also commonly called "error bars") to the plotted point to indicate the uncertainty associated with that point. We

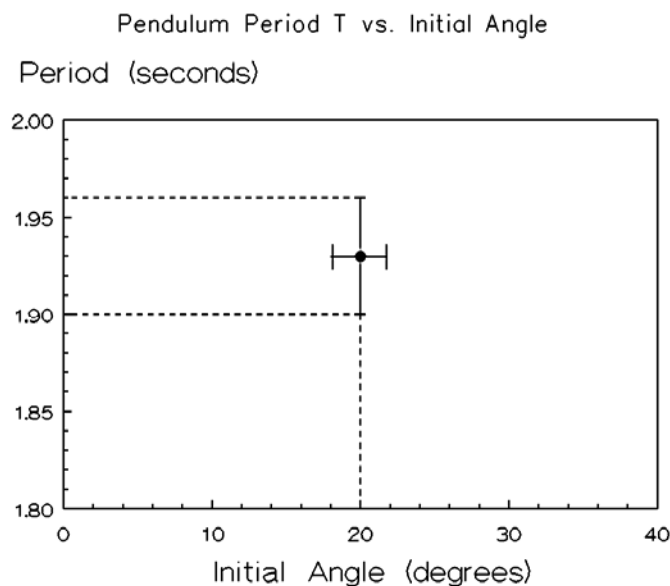

Figure 17: Plotted data points with uncertainty bars.
The point on the left has a y uncertainty of ± 1 and an x uncertainty of ± 2. The point on the right has a y uncertainty of ± 0.5 and an x uncertainty of ± 0.5.

do this by adding an "I-bar" to the graph, with the I-bar extending above and below the plotted

point (if the numerical axis is vertical). (If we are plotting a point on an (x, y) graph and there is some uncertainty in both the x and y values, then we use a horizontal I-bar for the x uncertainty and a vertical I-bar for the y uncertainty.

E.1.4   Rules for Significant Figures

The number of significant figures quoted for a given result should be consistent with the uncertainty in the measurement. In the previous example, it would be inappropriate to quote the results as 5 cm ± 0.1 cm (too few significant figures in the result) or as 5.132 cm ± 0.1 cm (too many significant figures in the result). Some scientists prefer to give the best estimate of the next significant figure after the one limited by the uncertainty, for example 5.13 cm ± 0.1 cm. The uncertainties themselves, since they are estimates, are usually quoted with only one significant figure, or in some cases, (for very high precision measurements, for example) with two significant figures. The following rules will determine how many significant figures there are:

1. All nonzero digits are significant. Ex.: 1-9.

2. Zeros between nonzero digits are significant. Ex.: 230504 (6 significant figures).

3. Leading zeros to left of nonzero digit are not significant. Such zeros only indicate position of decimal point. Ex.: 0.002 (1 significant digit).

4. Trailing zeros to right of decimal point are significant. Ex.: 0.0340 (3 significant digits).

5. Trailing zeros to the left of the decimal point may or may not be significant. Ex.: 50,600 (3,4 or 5 significant figures).

6. When adding or subtracting numbers, the final answer is round off to the decimal place equal to the number with the fewest decimals.

7. When multiplying or dividing numbers, the final answer is round off the same number of significant figures equal to the number with the fewest significant figures.

E.2   Systematic Errors, Precision and Random Effects

Why aren't measurements perfect? The causes of measurement uncertainties can be divided into three broad categories: *systematic errors*, *limited precision*, and *random effects*.

E.2.1   Systematic Errors

Systematic errors occur when a piece of equipment is improperly constructed, calibrated, or used. For example, suppose the stopwatch that you are using runs properly at 20° C, but you happen to be using it where the temperature is closer to 30° C, which (unknown to you) causes it to run 10% too fast.

One does not generally include systematic errors in the uncertainty of a measurement: if you know that a systematic problem exists, you should fix the problem (for example, by calibrating the stopwatch). The most appropriate thing to do with systematic problems in an experiment is to find them and to eliminate them. Unfortunately, no well-defined procedures exist for finding systematic errors: the best you can do is to be clever in anticipating problems and alert to trends in your data that suggest their presence. In some cases, you are aware of systematic effects but you are unable to determine their effects precisely. Then it would be appropriate to include in the stated measurement uncertainty a contribution due to the imprecise knowledge of the systematic effects.

E.2.2 Limited Precision

Limited precision is present because no measurement device can determine a value to infinite precision. Dials and linear scales (such as meter sticks, thermometers, gauges, speedometers, and the like) can *at best* be read to within one tenth (or so) of the smallest division on the scale. For example, the smallest divisions on a typical metric ruler are 1 mm apart: the *minimum* uncertainty for any measurement made with such a ruler is therefore about ± 0.1 mm. This last statement is not an absolute criterion, but a rule-of-thumb based on experience.

For measuring devices with a digital readout (a digital stopwatch, a digital thermometer, a digital voltmeter, and so on), the minimum uncertainty is ± 1 in the **last displayed digit**. For example, if your stopwatch reads 2.02 seconds, the "true" value for the time interval may range anywhere from 2.010...01 to 2.029999 seconds. (We don't know whether the electronics in the watch rounds up or down.) So, in this case, we must take the uncertainty to be at least ± 0.01 second.

For both digital and "analog" devices, we need to be aware of what is called **calibration uncertainty**. For example, a manufacturer may state that a particular voltmeter has been calibrated to ± 0.01 volt, that is, the readings of this meter agree with some standard, say, from the National Institute of Standards and Technology, to within 0.01 volt. Note that this uncertainty is different from the uncertainty in the displayed digits mentioned above. Although the manufacturer usually chooses the number of display digits so that the two types of uncertainty are about the same, that is not necessarily the case.

Accuracy and Precision. Calibration uncertainty is really an issue of what scientists call **accuracy**: how accurate is a particular measurement in terms of an accepted set of units such as volts, seconds, and so on? Accuracy is to be distinguished from **precision**, which is a question of the *internal* consistency and repeatability of a set of measurements without worrying about matching those measurements with other measurements or with standard units. For example, if I am using a stopwatch that is running too fast by a fixed amount, my set of timing measurements might be very *precise* if they are very repeatable. But they are not very *accurate* because my time units are not well matched to the standard second.

In both of the cases described above, these rules of thumb are meant to represent the *minimum* possible uncertainties for a measured value. Other effects might conspire to make measurements *more* uncertain than these limits, but there is nothing that you can do to make the uncertainties

smaller, short of buying a new device with a finer scale or more digits or a lower calibration uncertainty.

E.2.3   Random Effects

Random effects show up in the spread of results of repeated measurements of the same quantity. For example, five successive stopwatch measurements of the period of a pendulum might yield the results 2.02 s, 2.03 s, 2.01 s, 2.04 s, 2.02 s. Why are these results different? In this case, the dominant effect is that it is difficult for you to start and stop the stopwatch at *exactly* the right instant: no matter how hard you try to be exact, sometimes you will press the stopwatch button a bit too early and sometimes a bit too late. These unavoidable and essentially random measurement effects cause the results of successive measurements of the same quantity to vary.

In addition, the quantity being measured itself may vary. For example, as the temperature in the lab room increases and decreases, the length of the pendulum may increase and decrease. Hence, its period may change. In laboratory experiments, we try to control the environment as much as possible to minimize these fluctuations. However, if we are measuring the light radiated by a star, there is no way to control the star and its inherent fluctuations. Furthermore, fluctuations in the interstellar medium and in the earth's atmosphere may cause our readings to fluctuate in a random fashion.

In our analysis, we will assume that we are dealing with random effects, that is, we will assume that we have eliminated systematic errors. For most experiments we try to remove systematic errors and reduce calibration uncertainties and the effects of limited precision so that random effects are the dominant source of uncertainty.

E.3   Determining experimental uncertainties

There are several methods for determining experimental uncertainties. Here we mention three methods, which can be used easily in most of the laboratory measurements in this course.

E.3.1   Estimate Technique

In this method, we estimate the precision with which we can measure the quantity of interest, based on an examination of the measurement equipment (scales, balances, meters, etc.) being used and the quantity being measured (which may be "fuzzy," changing in time, etc.). For example, if we were using a scale with 0.1 cm marks to measure the distance between two points on a piece of paper, we might estimate the uncertainty in the measured distance to be about ±0.05 cm, that is, we could easily estimate the distance to within half a scale marking. Here we are estimating the uncertainty due to the limited precision or calibration uncertainty of our equipment.

E.3.2   Sensitivity Estimate

In this method, we estimate uncertainty by varying the the variable until we see a noticeable change. Thus, we are testing how sensitive the variable is to changes. For example, let's say you

are measuring the force due to a weight. You can add mass to the weight until you notice a change in the effect of the force. By doing this you can get a sense for how robust your system is to changes. Thus, we can estimate the uncertainty in the variable by slightly varying the variable and recording the range of values that lead to a null effect.

E.3.3 Repeated Measurement (Statistical) Technique

If a measurement is repeated in independent and unbiased ways, the results of the measurements will be slightly different each time. A statistical analysis of these results gives the "best" value of the measured quantity and the "best" estimate of the uncertainty to be associated with that result.

Mean Value (Average Value) The usual method of determining the best value for the result is to compute the "mean value" of the results: If x_1, x_2, ..., x_N are the N results of the measurement of the quantity x, then the mean value of x, usually denoted by $\overline{x}$, is defined as

$$\overline{x} = \frac{x_1 + x_2 + ... + x_N}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{48}$$

Standard Deviation The spread in the result is usually expressed as the "**root-mean-squared deviation**" (also called the "**standard deviation**"), usually denoted as σ_x (Greek letter sigma). Formally, the standard deviation is computed as

$$\sigma_x = \sqrt{\frac{(x_1 - \overline{x})^2 + ... + (x_N - \overline{x})^2}{N - 1}} \tag{49}$$

Let's decipher Eq. 49 in words. Eq. 49 tells us to take the difference between each of the measured values (x_1, x_2, ...) and the mean value $\overline{x}$. We then square each of the differences (so we count plus and minus differences equally). Next we find the average of the squared differences by adding them up and dividing by the number of measured values. (The -1 in the denominator of Eq. 49 is a mathematical refinement to reflect the fact that we have used the N values once to calculate the mean. In practice, the numerical difference between using N and using $N - 1$ in the denominator is insignificant for our purposes.) Finally, we take the square-root of that result to give a quantity which has the same units as the original x values. Figure 18 shows graphically the association of the standard deviation with the "spread" of x values in our set of measurements. Although determining the standard deviation may be tedious for a large array of data, it is generally accepted as the "best" estimate of the spread of the data.

Standard Deviation of the Mean (Standard Error) A further question arises: suppose we repeat sets of measurements many times and for each set we compute a mean value. How close do we expect the mean values to lie with respect to each other? Perhaps surprisingly, we can estimate this result from the data in a single run. Although we will not provide an explicit justification here (see John Taylor, *An Introduction to Error Analysis*, for example), the answer is given by the so-called **standard deviation of the mean** or the **standard error** :

$$\alpha = \frac{\sigma}{\sqrt{N}} \tag{50}$$

where N is the number of data points that go into the calculation of the mean and is the standard deviation for one "run." Note that we expect the *mean values* to be more closely clustered than the individual measurements.

In the scientific research literature, the standard deviation of the mean (or twice that for the 95% confidence range) is usually (but not always) the quantity cited for the uncertainty in the measured quantity.

Perhaps it is worthwhile emphasizing at this point that our analysis applies only to "random" uncertainties, that is, essentially uncontrollable fluctuations in equipment or in the system being measured, that collectively lead to scatter in our measured results. We have (implicitly) assumed that we have eliminated (or will correct for) so-called **systematic errors**, that is, effects that are present that may cause our results to be systematically (not randomly) high or low.

E.3.4 Interpretation of the Uncertainty

What meaning do we give to the uncertainty determined by one of the methods given above? The usual notion is that it gives us an estimate of the spread of values we would expect if we repeated the measurements many times (being careful to make the repetitions independent of one another). For example, if we repeated the distance measurements cited in the previous case study many times, we would expect most (statisticians worry a lot about how to make "most" more quantitative) of the measurements to fall within ±0.06 cm of each other.

95% Confidence Range   Sometimes uncertainties are expressed in terms of what is called the "95% Confidence Range" or "95% Confidence Limits." These phrases mean that if we repeat the measurements over and over many times, we expect 95% of the results to fall within the stated range. (It also means that we expect 5% of the results to fall outside this range!) Numerically, the 95% Confidence Range is about two times the standard deviation. Thus, we expect 95% of future measurements of that quantity to fall in the range centered on the mean value. Figure 18 shows the 95% confidence range for a particular distribution of values.

Asides: The standard deviation value is the "68% Confidence Range." The actual multiplicative factor for the 95% Confidence Range is 1.98 if the measurements are distributed according to the so-called "normal" ("Gaussian") distribution. But, for all practical purposes using 2 is fine for estimating the 95% Confidence Range.

> In general, we cannot expect exact agreement among the various methods of determining experimental uncertainties. As a rule of thumb, we usually expect the different methods of determining the uncertainty to agree within a factor of two or three.
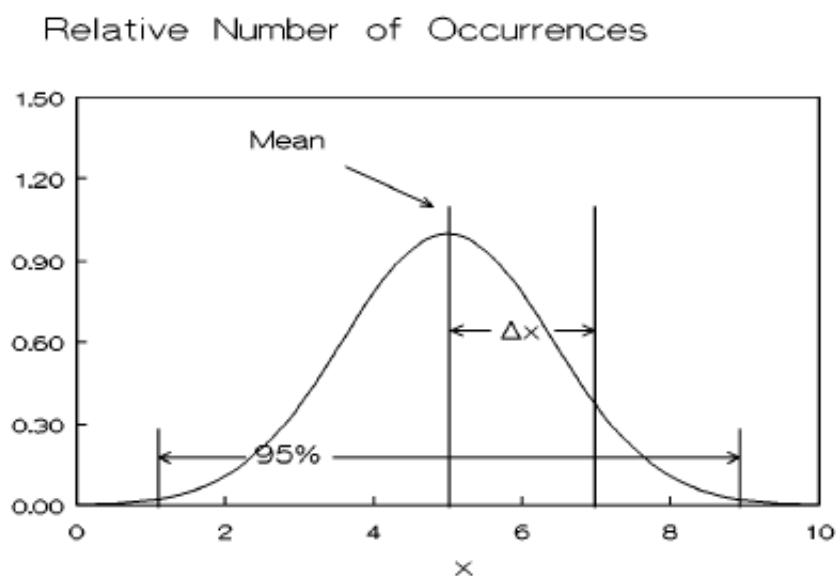
Figure 18: Distribution of x values from a set of measurements.
The mean value (average value) and standard deviation are indicated. The 95% Confidence
Range is shown.

Example Suppose that five independent observers measure the distance between two rather fuzzy marks on a piece of paper and obtain the following results:

$$
\begin{aligned}
d_1 &= 5.05 \text{ cm} \\
d_2 &= 5.10 \text{ cm} \\
d_3 &= 5.15 \text{ cm} \\
d_4 &= 5.20 \text{ cm} \\
d_5 &= 5.10 \text{ cm}
\end{aligned}
$$

If the observers were using a scale with 0.1 cm markings, the estimate technique would suggest an uncertainty estimate of about ±0.05 cm. The statistical technique yields a mean value $d = 5.12$ cm and for the standard deviation 0.057 cm ≈ 0.06 cm. We see that in this case we have reasonable agreement between the two methods of determining the uncertainties. We should quote the result of this measurement as 5.12 cm ± 0.06 cm or 5.12 cm ± 1%.

> In practice, it is not really meaningful to use the statistical estimation procedure unless you have at least ten or so independent measurements. For small data sets, you should simply estimate the uncertainty using one of the other methods cited above.

E.3.5 Assessing Uncertainties and Deviations from Expected Results

The primary reason for keeping track of measurement uncertainties is that the uncertainties tell us how much confidence we should have in the results of the measurements.

If the results of our measurements are compared to results expected on the basis of theoretical calculations or on the basis of previous experiments, we expect that, if no mistakes have been made, the results should agree with each other within the combined uncertainties. For the uncertainty we traditionally use the 95% Confidence Range (that is, two times the standard deviation).

Note: that even a theoretical calculation may have an uncertainty associated with it because there may be uncertainties in some of the numerical quantities used in the calculation or various mathematical approximations may have been used in reaching the result. As a rule of thumb, if the measured results agree with the expected results within the combined uncertainties, we usually can view the agreement as satisfactory. If the results disagree by more than the combined uncertainties, something interesting is going on and further examination is necessary.

Example Suppose a theorist from MIT predicts that the value of X in some experiment to be 333 ± 1 Nm/s. Suppose that an initial experiment gives the result 339 ± 7 Nm/s, which result overlaps the theoretical prediction within the combined uncertainties. Hence, we conclude that there is satisfactory agreement between the measured value and the predicted value given the experimental and theoretical uncertainties. However, suppose that we refine our measurement technique and get a new result 340.1 ± 0.1 Nm/s. Now the measured result and the theoretical result do <u>not</u> agree. [Note that our new measured result is perfectly consistent with our previous result with its somewhat larger uncertainty.] We cannot tell which is right or which is wrong without further investigation and comparison.

E.4 Propagating Uncertainties

This is sometimes called "propagation of errors". In most measurements, some calculation is necessary to link the measured quantities to the desired result. The question then naturally arises: How do the uncertainties in the measured quantities affect or propagate to the results? In other words, how do we estimate the uncertainty in the desired result from the uncertainties in the measured quantities?

E.4.1 "High-low Method"

One way to do this is to carry through the calculation using the extreme values of the measured quantities, for example 5.06 cm and 5.18 cm from the distance measurement example above, to find the range of result values. This method is straightforward but quickly becomes tedious if several variables are involved.

Example Suppose that you wish to determine a quantity, X, which is to be calculated indirectly using the measurements of a, b, and c, together with a theoretical expression: $X = \frac{ab}{c}$.
Suppose, further, that you have already determined that

$$a = 23.5 \pm 0.2 \text{ m}$$
$$b = 116.3 \pm 1.1 \text{ N}$$
$$c = 8.05 \pm 0.03 \text{ s}$$

The "best" value of X is computed from the best (mean) values of a, b and c:

$$X_{\text{best}} = \frac{23.5 \times 116.3}{8.05} = 339.509 \text{ Nm/s} \tag{51}$$

(We'll clean up the significant figures later.) But X could be about as large as what you get by using the maximum values of and b and the minimum (why?) value of c:

$$X_{\text{high}} = \frac{23.7 \times 117.4}{8.02} = 346.930 \text{ Nm/s} \tag{52}$$

And similarly, we find

$$X_{\text{low}} = \frac{23.3 \times 115.2}{8.08} = 332.198 \text{ Nm/s} \tag{53}$$

Notice that X_{high} and X_{low} differ from X_{best} by about the same amount, namely 7.3. Also note that it would by silly to give six significant figures for X. Common sense suggests reporting the value of X as, say, $X = 339.5 \pm 7.3$ Nm/s or $X = 339 \pm 7$ Nm/s.

E.4.2 General Method

The general treatment of the propagation of uncertainties is given in detail in texts on the statistical analysis of experimental data. A particular good reference at this level is John Taylor, *An Introduction to Uncertainty Analysis*. Here we will develop a very simple, but general method for finding the effects of uncertainties.

Suppose we want to calculate some result R, which depends on the values of several measured quantities x, y, z:

$$R = f(x, y, z) \tag{54}$$

Let us also suppose that we know the mean values and the uncertainties for each of these quantities. Then the uncertainty in R due to the uncertainty in x, for example, is calculated from

$$\Delta_x R = f(\overline{x} + \Delta\overline{x}, \overline{y}, \overline{z}) - f(\overline{x}, \overline{y}, \overline{z}) \tag{55}$$

where the subscript on Δ reminds us that we are calculating the effect due to x alone. We might call this the "partial uncertainty." Note that Eq. 55 is much like our "high-low" method except that we focus on the effect of just one of the variables. In a similar fashion, we may calculate the partial uncertainties in R due to Δy and to Δz.

By calculating each of these contributions to the uncertainty individually, we can find out which of the variables has the largest effect on the uncertainty of our final result. If we want to improve the experiment, we then know how to direct our efforts.

We now need to combine the individual contributions to get the overall uncertainty in the result. The usual argument is the following: If we *assume* that the measurements of the variables are independent so that variations in one do not affect the variations in the others, then we argue that the net uncertainty is calculated as the square root of the sum of the squares of the individual contributions:

$$\Delta R = \sqrt{(\Delta_x R)^2 + (\Delta_y R)^2 + (\Delta_z R)^2} \tag{56}$$

The formal justification of this statement comes from the theory of statistical distributions and assumes that the distribution of successive measurement values is described by the so-called **Gaussian** (or, equivalently, **normal**) distribution.

In rough terms, we can think of the fluctuations in the results as given by a kind of "motion" in a "space" of variables x, y, z. If the motion is independent in the x, y, and z directions, then the net "speed" is given as the square root of the sum of the squares of the "velocity" components. In most cases, we simply *assume* that the fluctuations due to the various variables are independent and use Eq. 56 to calculate the net effect of combining the contributions to the uncertainties.

Note that our general method applies no matter what the functional relationship between R and the various measured quantities. It is not restricted to additive and multiplicative relationships as are the usual simple rules for handling uncertainties.

The method introduced here is actually just a simple approximation to a method that uses partial derivatives. Recall that in computing partial derivatives, we treat as constants all the variables except the one with respect to which we are taking the derivative. For example, to find the contribution of x to the uncertainty in R, we calculate

$$\Delta_x R = \frac{\partial f(x, y, z)}{\partial x} \Delta x \tag{57}$$

with analogous expressions for the effects of y and z. We then combine the individual contributions as above.

Example Suppose we have made some measurements of a mass m, a distance r, and a frequency f, with the following results for the means and standard deviations of the measured quantities:

$$
\begin{aligned}
m &= 150.2 \pm 0.1 \\
r &= 5.80 \pm 0.02 \\
f &= 52.3 \pm 0.4
\end{aligned}
$$

(Note that we have omitted the units and hence lose 5 points on our lab report.) From these measured values we want to determine the "best value" and uncertainty for the following computed quantity: $F = mrf^2$. The "best value" is computed by simply using the best values of m, r, and f: $F = 2382875.2$ (We'll tidy up the number of significant figures later on.)

Let's use our partial derivative method to find the uncertainty. (As an exercise, you should compute the contributions to the uncertainty in F using the finite difference method of Eq. 55.) First, let's determine the effect due to m:

$$\Delta_m F = \frac{\partial F}{\partial m} \Delta m = r f^2 \Delta m = 1586. \tag{58}$$

Next, we look at the effect of r:

$$\Delta_r F = \frac{\partial F}{\partial r} \Delta r = m f^2 \Delta r = 8217. \tag{59}$$

And finally, the effect of f is given by

$$\Delta_f F = \frac{\partial F}{\partial f} \Delta f = 2 m r f \Delta f = 36449. \tag{60}$$

We see immediately that the measurement of f has the largest effect on the uncertainty of F. If we wanted to decrease the uncertainty of our results, we ought to work hardest at decreasing the uncertainty in f.

Finally, let's combine the uncertainties using the "square-root-of-the-sum-of-the-squares" method. From that computation we find that we ought to give F in the following form:

$$F = (2.383 \pm 0.037) \times 10^6 \tag{61}$$

or

$$F = (2.38 \pm 0.04) \times 10^6 \tag{62}$$

in the appropriate units. Note that we have adjusted the number of significant figures to conform to the stated uncertainty. As mentioned above, for most purposes, citing the uncertainty itself to one significant figure is adequate. For certain, high precision measurements, we might cite the uncertainty to two significant figures.

E.4.3 Connection to the Traditional Simple Rules for Uncertainties

To see where the usual rules for combining uncertainties come from, let's look at a simple functional form:

$$R = x + y \tag{63}$$

Using our procedure developed above, we find that

$$\Delta_x R = \Delta x$$
$$\Delta_y R = \Delta y$$

and combining uncertainties yields

$$\Delta R = \sqrt{(\Delta x)^2 + (\Delta y)^2} \tag{64}$$

The traditional rule for handling an additive relationships says that we should add the two (absolute) uncertainty contributions. We see that the traditional method overestimates the uncertainty to some extent.

E.5   Simplified Uncertainty Rules

E.5.1   Sum

$$\text{If } A = B + C \text{ then } \Delta A = \sqrt{(\Delta B)^2 + (\Delta C)^2} \tag{65}$$

E.5.2   Difference

$$\text{If } A = B - C \text{ then } \Delta A = \sqrt{(\Delta B)^2 + (\Delta C)^2} \tag{66}$$

E.5.3   Product

$$\text{If } A = B \times C \text{ then } \frac{\Delta A}{A} = \sqrt{(\frac{\Delta B}{B})^2 + (\frac{\Delta C}{C})^2} \tag{67}$$

E.5.4   Ratio

$$\text{If } A = \frac{B}{C} \text{ then } \frac{\Delta A}{A} = \sqrt{(\frac{\Delta B}{B})^2 + (\frac{\Delta C}{C})^2} \tag{68}$$

E.5.5   Multiplication by a Constant

$$\text{If } A = kB \text{ then } \Delta A = k\Delta B \tag{69}$$

E.5.6   Square Root

$$\text{If } A = \sqrt{B} \text{ then } \frac{\Delta A}{A} = \frac{1}{2}\frac{\Delta B}{B} \tag{70}$$

E.5.7   Powers

$$\text{If } A = B^n \text{ then } \frac{\Delta A}{A} = |n|\frac{\Delta B}{B} \tag{71}$$

E.5.8   Functions

$$\text{If } A = A(x) \text{ then } \Delta A = \left|\frac{dA}{dx}\right|\Delta x \tag{72}$$